stat teaser

ABOUT STAT-EASE SOFTWARE, TRAINING, AND CONSULTING FOR DOE
Phone 612.378.9449 Fax 612.378.2152 E-mail info@statease.com Web Site www.statease.com

Messing With Medieval Missile Machines (Part 2)

Beware to neighbors strolling with their babies by the Anderson estate!

Let me explain. It all started this past winter when I presented Stat-Ease's Experiment Design Made Easy workshop at the South Dakota School of Mines and Technology (SDSMT) in Rapid City. For show-and-tell they brought in a scale-model trebuchet issued to all new engineering students for hands-on experimentation. What a dream come true! I wasted no time trading a copy of our Design-Expert® software for one of these missile-flinging machines. (For more background on trebuchets, see Part 1 of this series in the June 2003 Stat-Teaser.) My goal was to make the trebuchet toss objects a given distance to fend off attackers (or door-to-door salespeople).

As soon as the snow melted off, I enlisted my son Hank as my gunnery mate. He can be seen in the picture above placing the missile (actually just a rubber ball) in a sling attached to a wooden arm via a fishing line. When the arm is released, the counterweights lever it upward, causing the ball to be flung forward 100 feet or more. (Unfortunately this exceeded the boundaries of the Anderson estate, thus putting innocent passersby in peril—but that didn't stop us!)

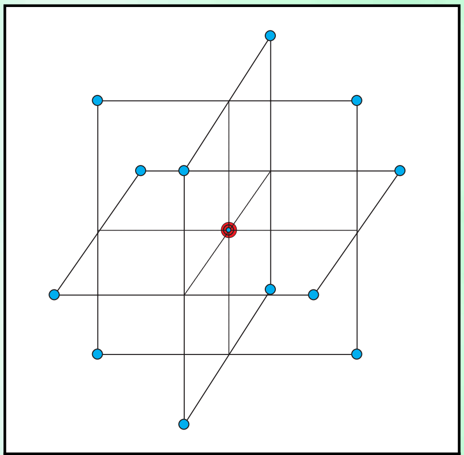

Figure 1: Box-Behnken design

The holes drilled through the wooden arm allow the weights and pivot point to be placed in various configurations. I decided to experiment on three key factors identified in prior studies by SDSMT students[1]:

A. Trebuchet arm length: 4 to 8 inches from the counterweight end to the point where the weights were hung
B. Counterweight: 10 to 20 pounds
C. Missile weight: 2 to 3 ounces

The missile consisted of a racquetball filled with varying amounts of salt. At first we tried heavier juggling balls (1 to

—*Continued on page 2.*

Workshop Schedule

DOE Simplified
November 20: Minneapolis, MN
An overview of Design of Experiments (DOE) from A to Z, based on the popular book. $295* ($195 each, 3 or more)

Statistics for Technical Professionals
February 18–19, 2004: Minneapolis, MN
Revitalize the statistical skills you need to stay competitive. $995* ($795 each, 3 or more)

Experiment Design Made Easy
October 21–23: Minneapolis, MN
November 18–20: Anaheim, CA
January 20–22, 2004: San Jose, CA
February 24–26, 2004: Minneapolis, MN
March 30–April 1, 2004: Philadelphia, PA
Study the practical aspects of DOE. Learn about simple, but powerful, two-level factorial designs. $1495* ($1195 each, 3 or more)

Response Surface Methods for Process Optimization
October 28–30: Minneapolis, MN
March 16–18, 2004: Minneapolis, MN
Maximize profitability by discovering optimal process settings. $1495* ($1195 each, 3 or more)

Mixture Design for Optimal Formulations
November 11–13: Minneapolis, MN
February 3–5, 2004: Minneapolis, MN
Find the ideal recipes for your mixtures with high-powered statistical tools. $1495* ($1195 each, 3 or more)

Robust Design: DOE Tools for Reducing Variability
April 13–15, 2004: Minneapolis, MN
Use DOE to create products and processes robust to varying conditions. A must for Six Sigma. *Factorial and RSM proficiency are required.* $1495* ($1195 each, 3 or more)

PreDOE: Basic Statistics for Experimenters
6-hour web-based training. This course or the equivalent is a prerequisite for all workshops—**www.statease.net**. $95

Attendance is limited to 20. Contact Sherry at **800.801.7191 x18** or **sherry@statease.com**.

**Includes a $95 student materials charge which is subject to state and local taxes.*

Stat-Ease
statistics made easy™

— *Continued from page 1.*

2.5 pounds), but much to our chagrin, these flew backward out of the sling. The lighter rubber raquetballs worked much better, flying far over the fence and into the street (pity the innocent infants in their perambulators!).

Aided by Design-Expert, we employed a response surface method (RSM) in the form of the Box-Behnken design (BBD). The geometry for the BBD is illustrated by Figure 1. The results, distance measured in feet, can be seen in Table 1 below. To make the exercise more realistic, we aimed the missiles at a play fort about 80 feet away. Standard order 9 (run number 14) actually hit the target. We never expected this to happen!

Notice from the last five runs in standard order, all done at center point levels, how precisely the trebuchet threw the racquetball. Because it was tricky to get an exact spotting for landings, the measurements were rounded to the nearest foot. The three results at 91 actually varied by a number of inches, but that isn't much at such a distance. It was truly awesome to watch Hank launch a missile and follow its soaring flight just below the high branches of our silver maples and barely above the fort to within the length of my shoe. The level of control over the trajectory was unbelievable!

Std	Run	A	B	C	Y
1	13	4	10	2.5	33
2	12	8	10	2.5	85
3	4	4	20	2.5	86
4	7	8	20	2.5	113
5	2	4	15	2	75
6	17	8	15	2	104
7	9	4	15	3	40
8	8	8	15	3	89
9	14	6	10	2	83
10	3	6	20	2	108
11	11	6	10	3	49
12	6	6	20	3	101
13	1	6	15	2.5	88
14	5	6	15	2.5	91
15	15	6	15	2.5	91
16	10	6	15	2.5	87
17	16	6	15	2.5	91

Table 1: Design Information

ANOVA for Response Surface Quadratic Model					
Source	Sum of Squares	DF	Mean Square	F Value	Prob > F
Model	8113.02	9	901.45	229.88	< 0.0001
A	3081.13	1	3081.13	785.71	< 0.0001
B	3120.50	1	3120.50	795.76	< 0.0001
C	1035.13	1	1035.13	263.97	< 0.0001
A^2	364.17	1	364.17	92.87	< 0.0001
B^2	4.64	1	4.64	1.18	0.3126
C^2	45.85	1	45.85	11.69	0.0111
AB	156.25	1	156.25	39.85	0.0004
AC	100.00	1	100.00	25.50	0.0015
BC	182.25	1	182.25	46.48	0.0002
Residual	27.45	7	3.92		
Lack of Fit	12.25	3	4.08	1.07	0.4543
Pure Error	15.20	4	3.80		
Cor Total	8140.47	16			

Table 2: ANOVA for the trebuchet model

As shown in Table 2 the relatively small process variation, as compared to the changes induced by the controlled factors, led to a highly significant quadratic model. The residuals exhibited no aberrant patterns on the diagnostic plots (normal plot, residuals versus predicted level, etc.).

Figure 2 shows the contour plot and corresponding 3D view for Factor A (arm length) versus Factor B (counterweight) while holding Factor C (missile weight) at its center point value (2.5 ounces). Notice that the contours for 75 to 85 feet cut through the heart of the graph on a diagonal. This range represents the operating window or "sweet spot." It puts the 2.5 ounce missile somewhere on the target at a broad array of A-B setup combinations. The red points on the graph represent actual settings.

A compelling view of the sweet spot is

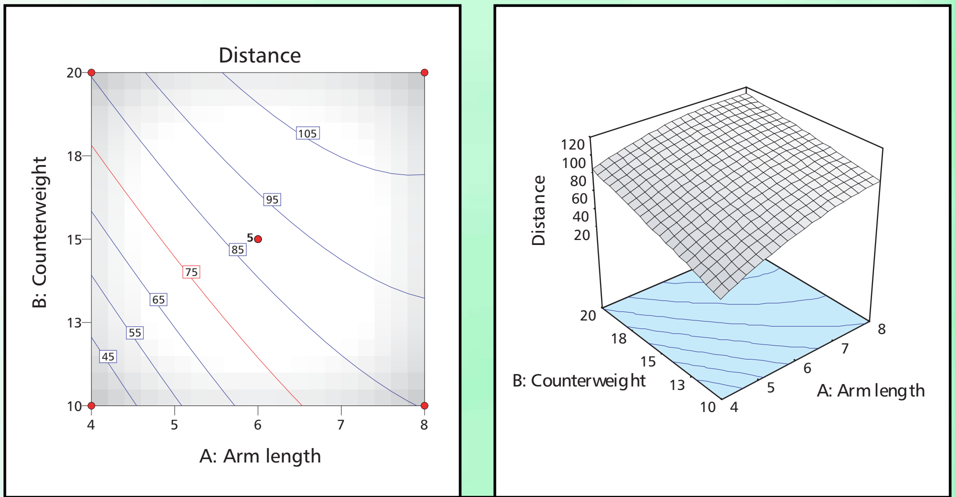

Figure 2: Response surface plots—2D and 3D

Choosing the Right RSM Design

Ever wonder what the difference is between the various response surface method (RSM) design options? To help you choose the best design for your DOE, I've put together a list of things you should know about each of the three primary response surface designs—Central Composite, Box-Behnken, and D-optimal.

Central Composite Design (CCD)

- Was developed for estimating a quadratic model
- Is created from a two-level factorial design, and augmented with center points and axial points
- Is rather insensitive to missing data
- Has five levels for each factor (Note: The number of levels can be modified by choosing alpha=1.0, a face-centered CCD which has only three levels for each factor.)
- Provides excellent prediction capability near the center of the design space (where the presumed optimum is if it includes a replicated center point as specified)

Box-Behnken Design (BBD)

(See Figure 1 on page 1.)

- Was created for estimating a quadratic model
- Always has 3 levels for each factor
- Has specific positioning of design points (For example, this allowed Mark to do the Trebuchet RSM with pre-drilled holes.)
- Provides strong coefficient estimates near the center of the design space (where the presumed optimum is), but is weaker at the corners of the cube (where there aren't any design points)

Other comments: If you end up missing any runs, the accuracy of the remaining runs becomes critical to the dependability of the model. (I don't recommend BBD's if it is common to have a bad run or missing data. The central composite designs have more runs initially and this makes them more robust to problems.)

D-Optimal Design

- Can be used to create a good design for fitting a linear, quadratic or cubic model (Note: In Design-Expert® software you can change the user preferences to get up to a 6th order model.)
- May have 3-4 levels in quadratic models
- Positions design points mathematically according to the number of factors and the desired model, therefore the points are not at any specific positions—they are simply spread out in the design space to meet the d-optimality criteria (see below)
- Chooses points to minimize the integrated variation of the coefficients for the model, giving you the most precise coefficients

Other comments: If you have knowledge of the subject matter, you can edit the desired model by removing the terms that you know are not significant or can't exist. This will decrease the required number of runs. You can also add constraints to your design space, for instance to exclude a particular area where you can't get a response.

Hopefully this information will help you choose the right RSM design. For an in-depth exploration of response surface methods, attend Stat-Ease's Response Surface Methods for Process Optimization workshop. For more information, see page 4.

Shari@StatEase.com

— Continued from page 2.

provided by the "graphical overlay" plot shown in Figure 3 at a different slice for factor C—its lower level (2.0 ounce missile). The flag shows the setup used at standard order 9 listed on Table 1, which actually hit the fort as predicted. (I chose this slice because it includes an actual setting used. A sweet spot appears at any weight between 2.0-3.0 ounces but the trebuchet has limited settings due to the location of the drill holes. In theory we could drill new holes to achieve other settings, but...)

Hank and I were really excited about actually hitting something on purpose! Now that we've mastered the trebuchet,

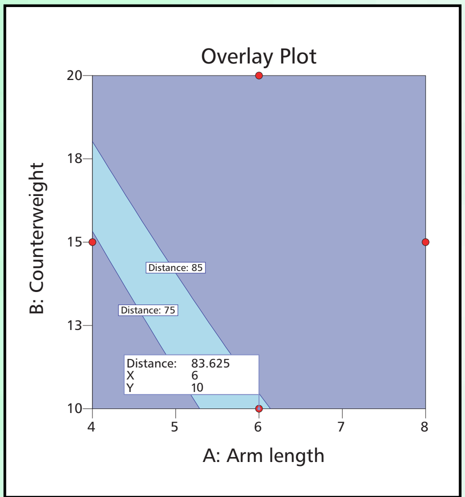

Figure 3: Graphical overlay plot

the neighbors, at least the ones whom we like, need not fear unexpected bombardments while strolling by the Anderson estate. They've started coming by again but it's funny to see people walking (not riding) with bicycle helmets on their heads. There's been no sign yet of any baby strollers.

Mark@StatEase.com

[1]Burris, et al. Trebuchet report by team "Hurl." General engineering project for Professor David Dixon, South Dakota School of Mines and Technology, December 17, 2002.

Learn How to Optimize your Process with RSM!

9/03

In design of experiments, response surface methods (RSM) are used to optimize product and process performance. RSM helps you define the precise settings necessary to achieve your goals, whether it be to produce high quality products, lower costs, reduce defects, or be like Mark and hit a play fort! Stat-Ease's Response Surface Methods for Process Optimization workshop provides hands-on knowledge of these powerful tools. This computer-intensive, intermediate-level three-day class is recommended for anyone seeking a competitive edge. Knowledge of factorial designs is a prerequisite, which can be gained from the Experiment Design Made Easy (EDME) workshop.

The RSM workshop covers the in's and out's of central composite designs and how they can be modified to meet your design criteria. In this class, you will also look at D-optimal, Box-Behnken, Three-Level Factorial, and Hybrid designs. Because D-optimal designs provide maximum design flexibility, they will be used to illustrate common experimental problems. You will learn how to add categorical factors and impose linear constraints on a design space. In addition, you will discover how to use the powerful design evaluation features in Design-Expert software to determine if a design will meet your needs, before you actually perform the runs.

Analysis of response surface designs will be covered in detail, including how and when to use backward, forward and stepwise algorithmic model reduction. You will also learn how to create customized contour and 3D surface graphs that clearly illustrate the "sweet spot" in your process.

The RSM workshop covers numerical and graphical optimization routines that are favorites with experimenters. You'll discover how to maximize the information gained from these tools, plus a few tricks you probably haven't thought of yet! If you've mastered factorial design, it is time to expand your knowledge by learning RSM methods.

Join us in Minneapolis, MN on Oct 28-30 for our next Response Surface Methods for Process Optimization workshop.

If you have at least 4-6 students who are already versed in factorial design, consider bringing the RSM workshop in-house. Contact Sherry Klick at **612.378.9449 x18** for a quotation.

Those of you working with formulations or mixtures should take a look at our Mixture Design for Optimal Formulations (MIX) workshop. This course is equivalent to the RSM workshop, but it explores mixture designs and mixture constraints. Information on all workshops is available at **http://www.statease.com/training.html**.

Stat-Ease®
statistics made easy™
Stat-Ease, Inc., Hennepin Square
Suite 480, 2021 E. Hennepin Ave.
Minneapolis, MN 55413-2726
Address Service Requested

Presorted
Standard
U.S. POSTAGE PAID
Minneapolis, MN
Permit No. 28684