Problem 4-2 (as stated in *RSM Simplified*)

This is a continuation of the experiment on confetti started in the previous chapter (originally reported in chapter 8 of DOE Simplified). ANOVA on the results from the two-level factorial design with center points reveals significant curvature. Therefore the experimenters add a second block of runs to create a CCD. The result can be seen in Table 4-7.

Table 4-7. Second block of runs to optimize flight time of confetti

Std	*A: Width (inches)*	*B: Length (inches)*	*Time (seconds)*
9	*0.60*	*4.00*	*2.5*
10	*3.40*	*4.00*	*1.8*
11	*2.00*	*2.60*	*2.6*
12	*2.00*	*5.40*	*3.0*
13	*2.00*	*4.00*	*2.5*
14	*2.00*	*4.00*	*2.6*
15	*2.00*	*4.00*	*2.6*
16	*2.00*	*4.00*	*2.9*

Points 9 through 12 in standard (Std) order represent the stars placed approximately 1.4 coded units from the center along the two axes A and B. As you can decipher from the factor levels, the remaining four runs are center points that help to link this second block to the first (shown on Table 3-8). Analyze the entire set of data for confetti and determine the source of the curvature in response. Then find the optimal configuration of length and width (within the experimental bounds) to maximize flight time.

Solution to Problem 4-2

The CCD contains five levels of each factor: low axial, low factorial, center, high factorial and high axial. With this many levels, it generates enough information to fit a second order polynomial called a "quadratic." Standard statistical software can do the actual fitting of the model. The quadratic model for confetti flight time is:

$$\text{Time} = 2.68 - 0.30A + 0.12B - 0.050AB - 0.31\,A^2 + 0.020B^2$$

This model is expressed in terms of the coded factor levels (–1 for low and +1 for high, based on the factorial core of the CCD). The coding eliminates problems caused by varying units of measure, such as inches versus centimeters, which can create problems when comparing coefficients. In this case, the A-squared (A^2) term has the largest coefficient, which indicates curvature along this dimension. The ANOVA, shown in Table 4-2.1, indicates a high degree of significance for this term and the model as a whole. The AB and B^2 terms are not significant, but there is no benefit to eliminating them from the model because the response surface will not be affected.

Table 4-2.1: ANOVA for CCD on confetti

Source	Sum of Squares	DF	Mean Square	F Value	p-value Prob > F
Block	0.016	1	0.016		
Model	1.60	5	0.32	15.84	0.0003
A	*0.72*	*1*	*0.72*	*35.47*	*0.0002*
B	*0.12*	*1*	*0.12*	*5.77*	*0.0397*
A^2	*0.75*	*1*	*0.75*	*37.37*	*0.0002*
B^2	*0.003*	*1*	*0.003*	*0.15*	*0.7031*
AB	*0.010*	*1*	*0.010*	*0.50*	*0.4991*
Residual	0.18	9	0.020		
Lack of fit	*0.071*	*3*	*0.024*	*1.30*	*0.3578*
Pure Error	*0.11*	*6*	*0.018*		
Cor Total	1.79	15			

Lack of fit is not significant (because the probability value of 0.3578 exceeds the threshold value of 0.05). Also, diagnosis of residuals showed no abnormality. Therefore, the model is statistically solid. The resulting contour graph is shown in Figure 4-2.1.

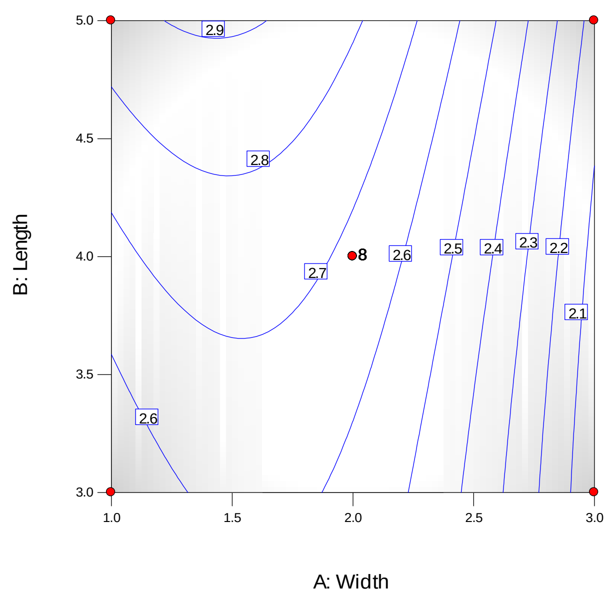

Figure 4-2.1: Contour graph for confetti flight time

Each contour represents a combination of input factors that produces a constant response, as shown by the respective labels. The actual runs are shown as dots. (The number 8 by the center point indicates the number of replicates at this set of conditions. In other words, at eight random intervals throughout the experiment, we reproduced confetti with the mid-point dimensions of 2 by 4 inches.) Normally we would restrict the axes to the factorial range to avoid extrapolation beyond the experimental space, but we wanted to show the entire design. Notice the darkening in areas outside of the actual design space, especially in the corners. These represent areas where predictions will be unreliable due to lack of information. Figure 4-2.2 shows a 3-D response surface with the ranges reduced to their proper levels.

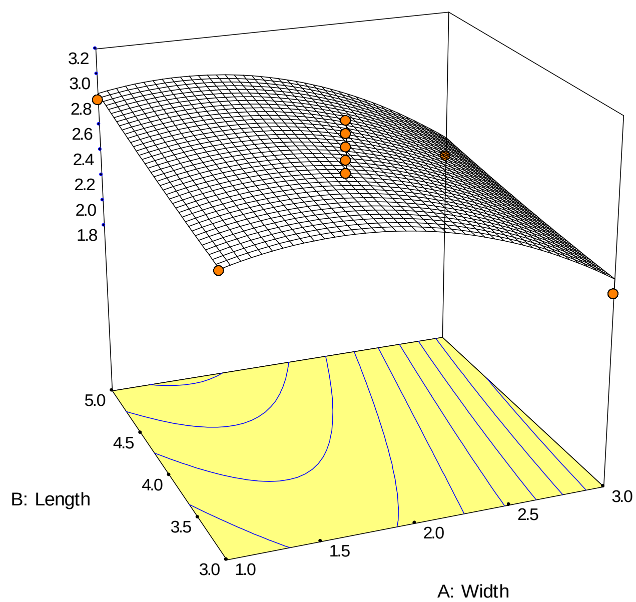

Figure 4-2.2.: Response surface for confetti flight time

Notice how the surface curves along the A-dimension (length), but not by B (width). The maximum flight time within this factorial range occurs at a width of 1.44 inches and length of 5 inches. Longer confetti might fly even longer, but this would need to be determined via further experimentation.