Problem 3-2 (as stated in *RSM Simplified*)

In chapter 8 of DOE Simplified we describe an experiment on confetti made by cutting strips of paper to the following dimensions:

A. *Width, inches: 1 to 3.*

B. *Height, inches: 3 to 5.*

The two-level design with center points is shown below with the resulting flight times.

Table 3-9. Flight times for confetti made to varying dimensions

Std	*A: Width (inches)*	*B: Length (inches)*	*Time (seconds)*
1	1.00	3.00	2.5
2	3.00	3.00	1.9
3	1.00	5.00	2.8
4	3.00	5.00	2.0
5	2.00	4.00	2.8
6	2.00	4.00	2.7
7	2.00	4.00	2.6
8	2.00	4.00	2.7

Each strand was dropped 10 times and then the results were averaged. These are considered to be duplicates rather than true replicates because some operations are not repeated, such as the cutting of the paper. On the other hand we show four separate runs of center points (standard orders 5-8), which were replicated by repeating all process steps, that is, re-cutting the confetti to specified dimensions four times, not simply re-dropping the same piece four times. Thus, the variation within center points provides an accurate estimate of the pure error of the confetti production process. Set up and analyze this data. Do you see significant curvature? (To check your results, see chapter 8 of DOE Simplified.)

Solution to Problem 3-2

Figure 3-2.1 shows where the design points are located.

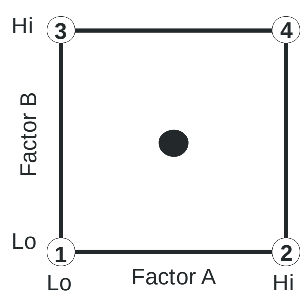

Figure 3-2.1: Two-level factorial design with center point

The two-level design with center point (pictured in Figure 3-2.1) requires all factors to be set at their mid-level, around which you run only the combinations of the extreme lows and highs. It differs from a full three-level factorial, which would require nine distinct combinations, including points at the midpoints of the edges. The two-level factorial with center points will reveal curvature in your system, but it does not provide the complete picture that would be obtained by doing the full three-level factorial.

The response is flight time in seconds from a height of five feet. The half-normal plot of effects for this data is shown in Figure 3-2.2.

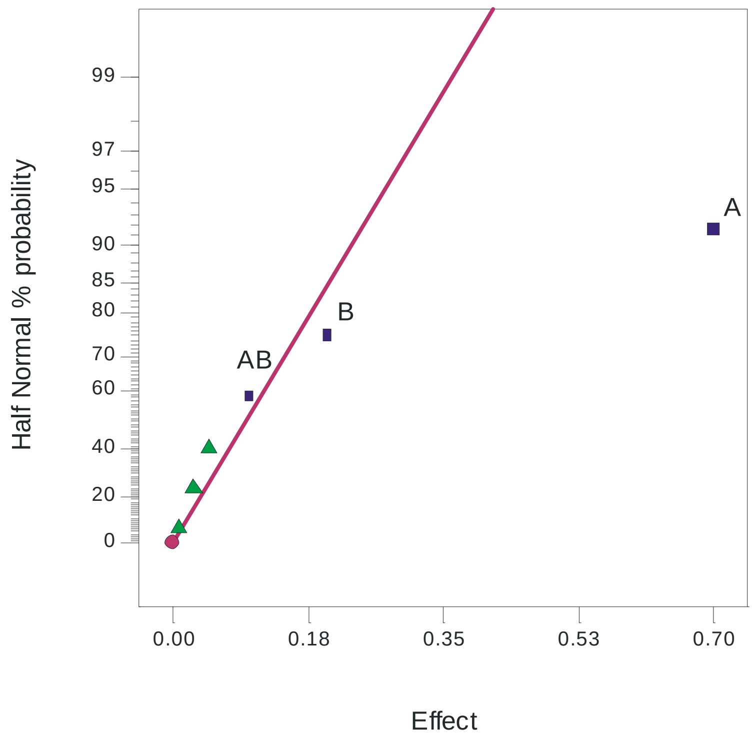

Figure 3-2.2: Half-normal plot of effects for confetti experiment

Factor A, the width, stands out as a very large effect. On the other end of the effect scale (nearest zero) notice the three triangular symbols. These come from the replicated center points, which contribute three degrees of freedom for estimation of “pure error.” In line with the pure error, you will find the main effect of B (length) and the interaction AB. These two relatively trivial effects will be thrown into the residual pool under a new label, “lack of fit,” to differentiate these estimates of error from the “pure error.” The pure error is included in the residual subtotal in the ANOVA, shown in Table 3-2.1, which also exhibits a new row called “curvature.”

Table 3-2.1: ANOVA for confetti experiment (effects B and AB used for lack of fit test)

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	0.49	1	0.49	35.00	0.0020
A	*0.49*	*1*	*0.49*	*35.00*	*0.0020*
Curvature	0.32	1	0.32	22.86	0.0050
Residual	0.070	5	0.014		
Lack of fit	*0.050*	*2*	*0.025*	*3.75*	*0.1527*
Pure error	*0.020*	*3*	*0.0067*		
Cor Total	0.88	7			

Apply the usual 0.05 rule to assess the significance of the curvature. In this case, the probability value of 0.005 for curvature falls below the acceptable threshold of 0.05, so it cannot be ignored. That's bad. It means that the results at the center point were unexpectedly high or low relative to the factorial points around it. Figure 3-2.3 shows effect plots of the response versus factors A and B.

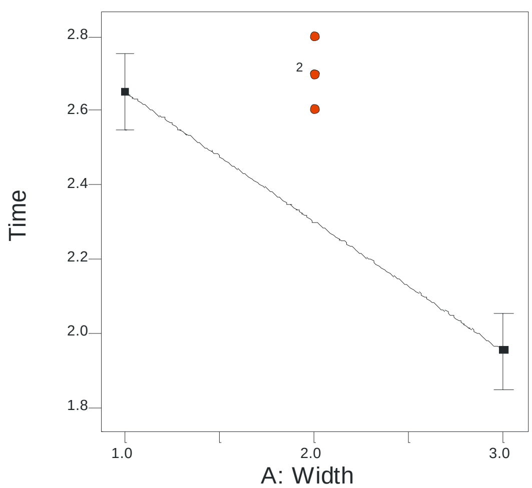

Figure 3-2.3a: Effect plot for width (A)

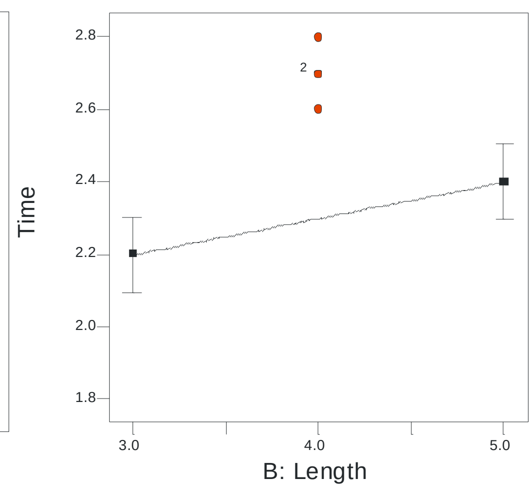

Figure 3-2.3b: Effect plot for Length (B)

The relationships obviously are not linear. Notice that the center point responses stay the same in both plots. Because all factors are run at their mid-levels, we cannot say whether the observed curvature occurs in the A or the B direction, or some of both. Statisticians express this confusion as an alias relationship: Curvature = $A^2 + B^2$. It will take more experimentation to pin this down. The next step is to augment the existing design via response surface methods (RSM).