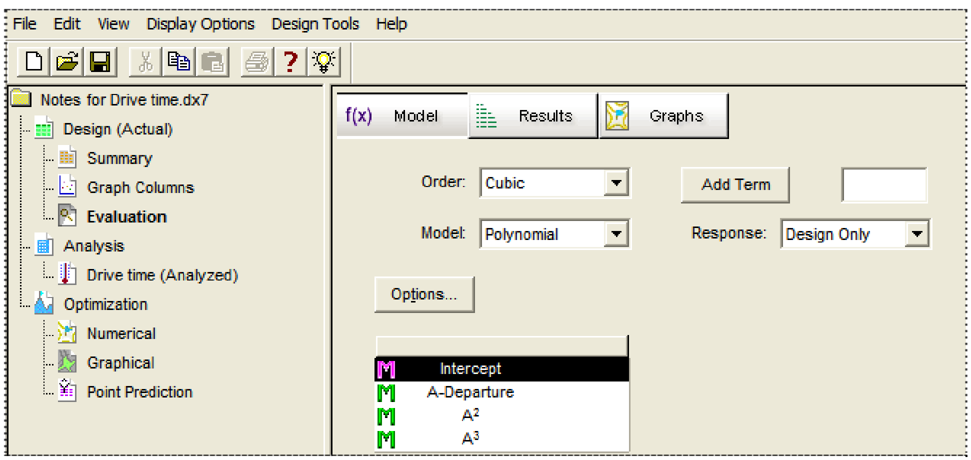
One-Factor RSM Tutorial (Part 2 – Advanced topics)

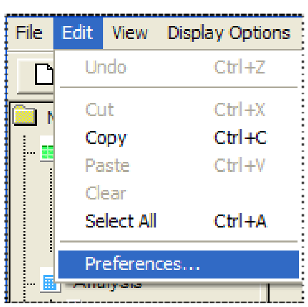
Adding Higher-Order Model Terms

If you still have the driving data active in Design-Expert® software from Part 1 of this tutorial, continue on. If you exited the program, re-start it and use **File**, **Open Design** to open your data file (**Driving.dx7**). Otherwise, go back and set it up as instructed in One-Factor RSM Tutorial (Part 1 – The Basics). The wavy curve you see on the response surface plot for drive time is characteristic of a third-order (cubic) polynomial model. Could an even higher-order model be applied to the data from this case? If so, would it improve the fit? Under the **Design** branch click the **Evaluation** node.



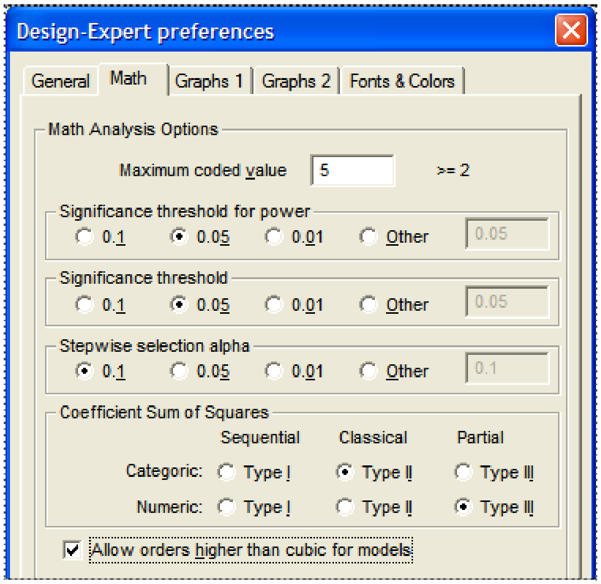
Design evaluation

Now select **Edit**, **Preferences**. The settings there must be changed to allow models of higher-order than cubic.



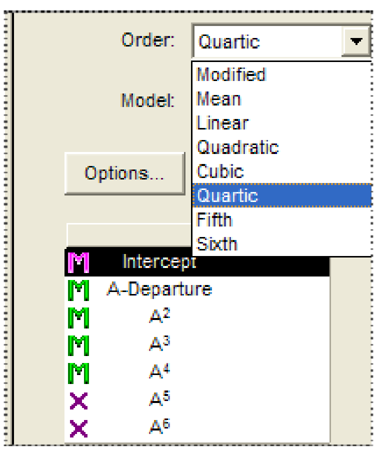
Edit preferences selection

Click the **Math** tab and turn on the option to **Allow orders higher than cubic for models** shown at the bottom of the dialog box.



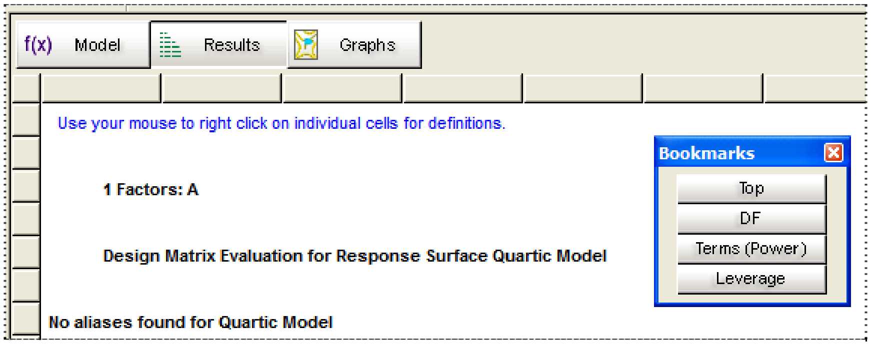
Math preferences – enabling high-order models

Feel free to look over all the other Preferences tabs: General, Graphs 1, Graphs 2 and Fonts & Color. Press **OK** to return to the design evaluation screen. Change the **Order** to **Quartic** or double-click the term A^4 to put it in the model ("M").



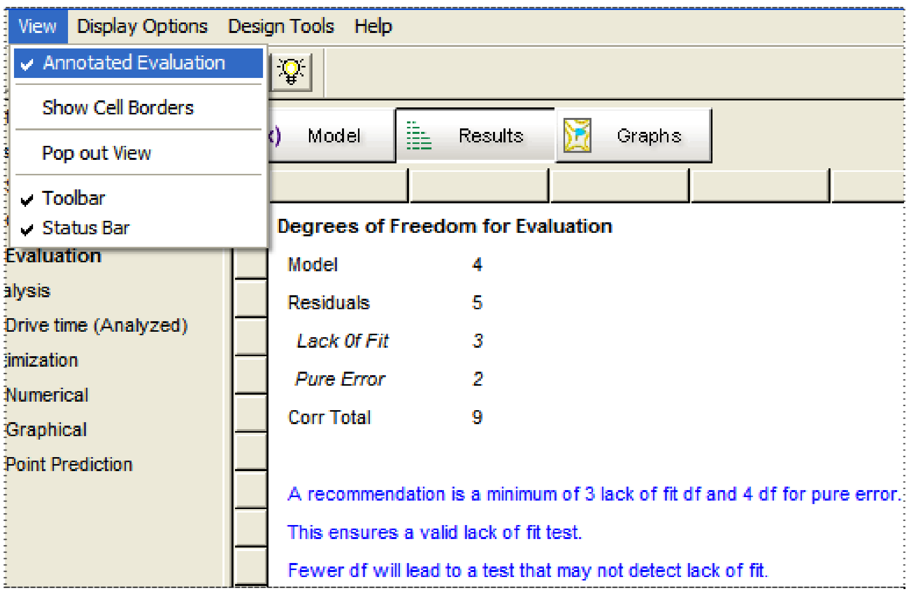
Model changed to quartic (4^{th} order)

Click ***Results*** to see the evaluation of this higher-order model.



Evaluation finds no aliases for quartic model

No aliases are found, but other aspects of the evaluation fall short of the ideal. <u>Scroll down</u> the output (or use the Bookmarks) and pay close attention to the annotations. If you do not see these on-screen, go to View and select Annotated Evaluation.



Annotated view – degrees of freedom detailed

<u>Scroll down</u> further to see statistics on power. Reading the annotations below, you will realize that going to the quartic model may not be such a good idea.

				Power at 5 % alpha level for effect of		
Term	StdErr**	VIF	Ri-Squared	0.5 Std. Dev.	1 Std. Dev.	2 Std. Dev.
A	1.56	15.78	0.9366	5.2 %	5.8 %	8.3 %
A^2	2.14	14.15	0.9293	5.4 %	6.7 %	12.0 %
A^3	1.78	29.86	0.9665	5.2 %	5.6 %	7.5 %
A^4	1.69	28.30	0.9647	5.7 %	7.8 %	16.3 %

**Basis Std. Dev. = 1.0

Standard errors should be similar within type of coefficient. Smaller is better.

Ideal VIF is 1.0. VIF's above 10 are cause for alarm,
indicating coefficients are poorly estimated due to multicollinearity.

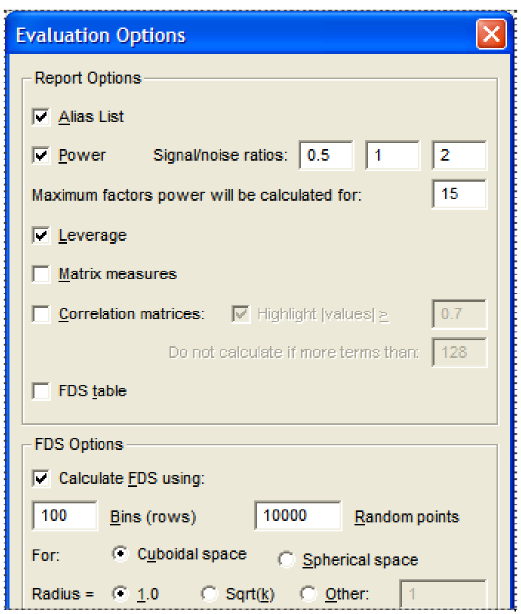
Ideal R-squared is 0.0. High R-squared means terms are correlated with each other,
possibly leading to poor models.

Power should be approximately 80% for the effect you want to detect.

Power statistics

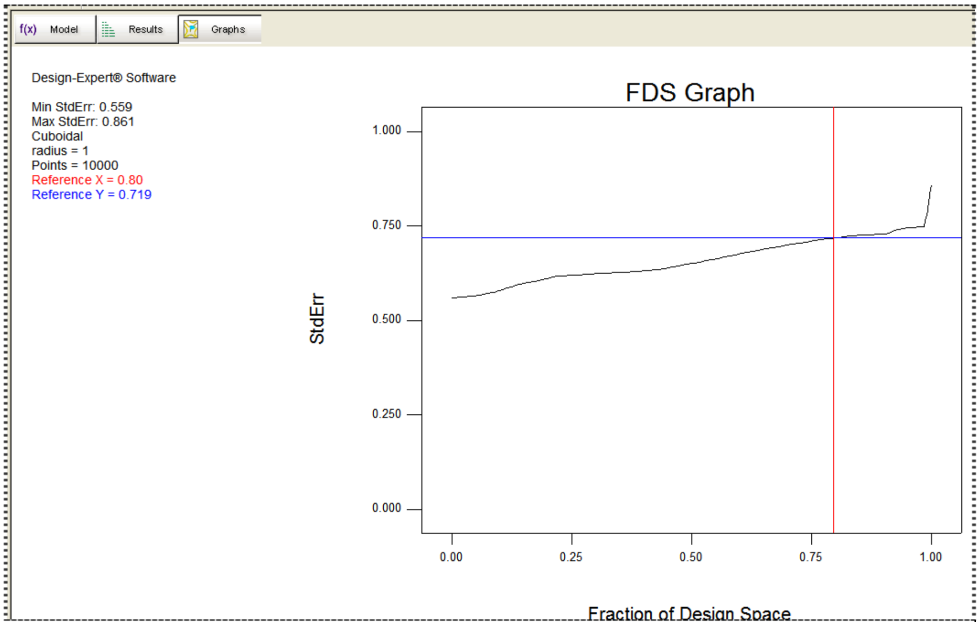
Scroll down to the bottom of this report and view the leverages. Note the design point with the unusually high leverage of 0.9743. This is the late departure time near 50 minutes that occurred due to Mark oversleeping. This is what experts on DOE refer to as a 'botched' factor setting. It should not be surprising to come out poorly for leverage.

Believe it or not, many more evaluation statistics can be generated from Design-Expert if you like – the ones shown by default are those most easily digested by non-statisticians. To enable additional measures and modify defaults click the Options under the Model screen.



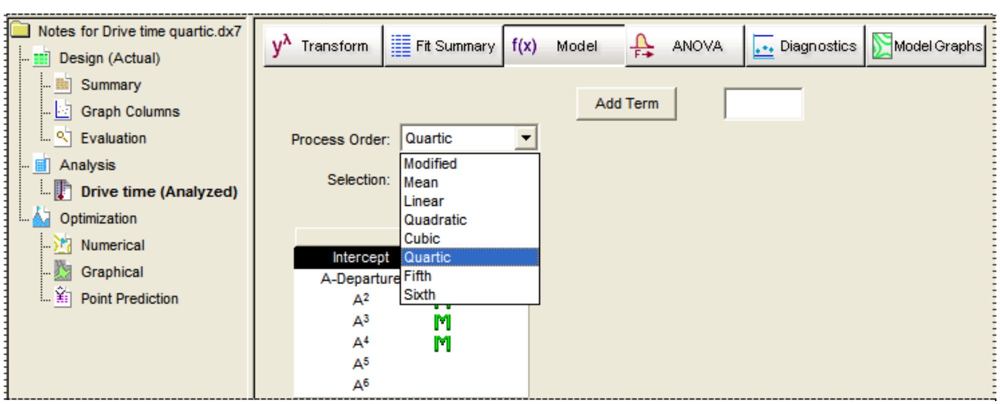
Evaluation options

Press ahead to the Graphs to see the plot of FDS – fraction of design space. Click on the curve of standard error at a fraction near 0.8 (80 percent) to generate cross-reference lines like those shown in the screen shot below.



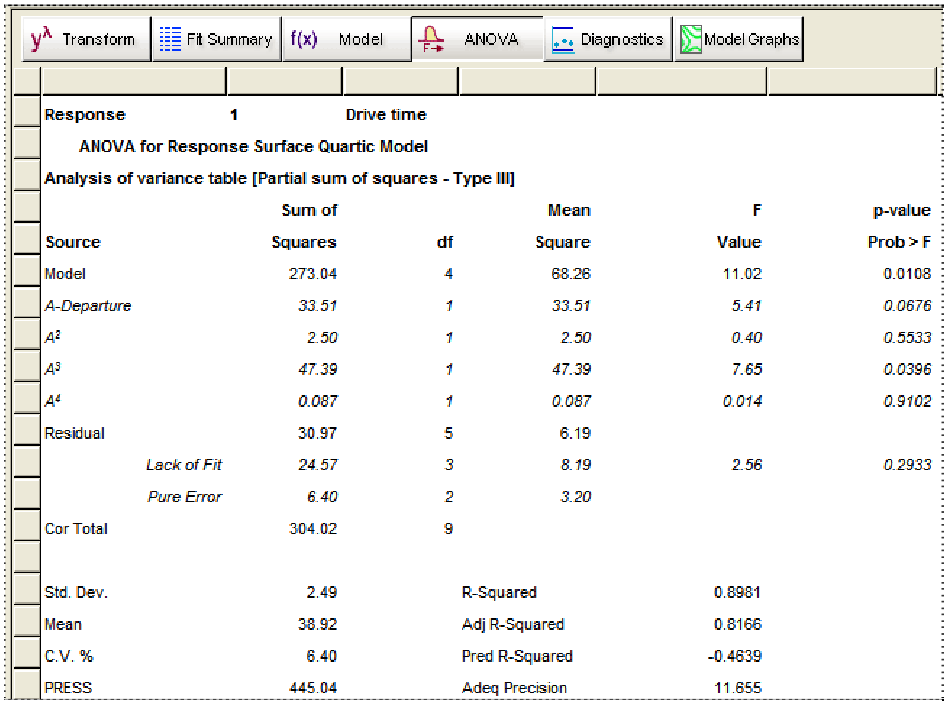
FDS graph

As noted in Screen Tips (hint: press the light-bulb icon), this is a line graph showing the relationship between the "volume" of the design space (area of interest) and amount of prediction error. The curve indicates what fraction (percentage) of the design space has a given prediction error or lower. In general, a lower and flatter FDS curve is better. The FDS graph, suggested to us by DOE guru Douglas Montgomery, provides very helpful information on scaled prediction variance (SPV) for comparing alternative test matrices – simple enough that even non-statisticians can see differences at a glance and versatile for any type of experiment – mixture, process or combined. For example, one could re-do the FDS graph for the cubic model and compare results and/or try some other experiment designs. However, let's not belabor the evaluation: Go back to the **Analysis** branch and click the **Drive time** node. Then press ahead to the ***Model*** and change the order there to **Quartic**.



Changing model to quartic for analysis

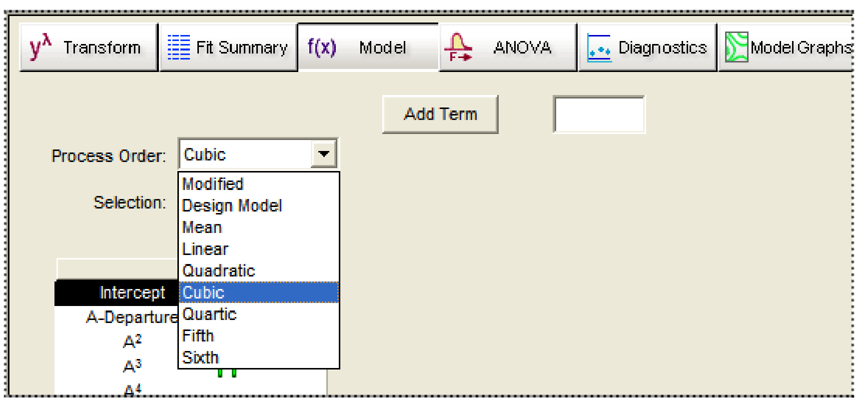
Then click the ***ANOVA*** button. Notice that not only does the A^4 term come out insignificant (p-value of 0.91), but the Pred R-Squared goes negative – not a good sign!



Response	**1**	**Drive time**			
ANOVA for Response Surface Quartic Model					
Analysis of variance table [Partial sum of squares - Type III]					
	Sum of		**Mean**	**F**	**p-value**
Source	**Squares**	**df**	**Square**	**Value**	**Prob > F**
Model	273.04	4	68.26	11.02	0.0108
A-Departure	*33.51*	*1*	*33.51*	*5.41*	*0.0676*
A^2	*2.50*	*1*	*2.50*	*0.40*	*0.5533*
A^3	*47.39*	*1*	*47.39*	*7.65*	*0.0396*
A^4	*0.087*	*1*	*0.087*	*0.014*	*0.9102*
Residual	30.97	5	6.19		
Lack of Fit	*24.57*	*3*	*8.19*	*2.56*	*0.2933*
Pure Error	*6.40*	*2*	*3.20*		
Cor Total	304.02	9			
Std. Dev.	2.49	R-Squared	0.8981		
Mean	38.92	Adj R-Squared	0.8166		
C.V. %	6.40	Pred R-Squared	-0.4639		
PRESS	445.04	Adeq Precision	11.655		

ANOVA for quartic model (annotations turned of in View menu)

Before moving on to the next topic, return to the Model button and re-set the Order to Cubic.



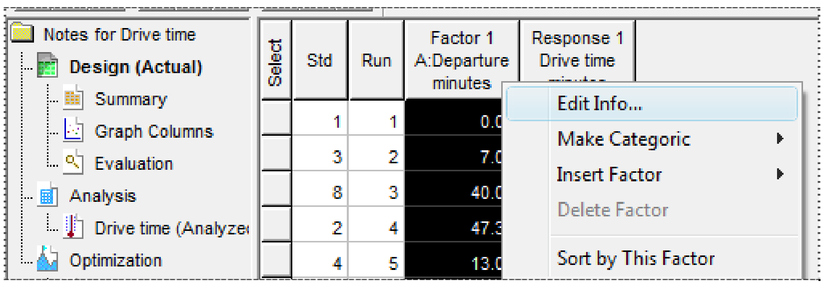
Back to the cubic model

By the way, Design-Expert knows enough in this simplistic one-factor case to add up to sixth order terms to the model list. However, in some cases, you may need to make use of the Add Term entry field. For example, in a two-factor RSM you can add terms such as A^2B^4 or A^3B^2.

Propagation of Error (POE)

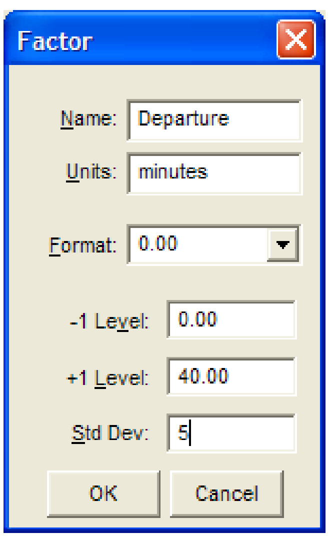
Seeing such a rapid increase in drive time predicted for late departures makes Mark more aware of how much the response depends on what time he leaves home. He realizes that a 5 minute deviation one way or the other would not be an unreasonable expectation. How will this cause the drive time to vary? Perhaps by aiming for a specific departure Mark might reduce drive time variation caused by day-to-day differences in when he heads off for work. Via its capability to calculate and plot propagation of error (POE), Design-Expert can provide enlightenment on these issues.

Click the Design branch to bring up the run-sheet for the experiment. Then right-click the column-header for the **Factor 1 (**A:Departure) and select **Edit Info**.



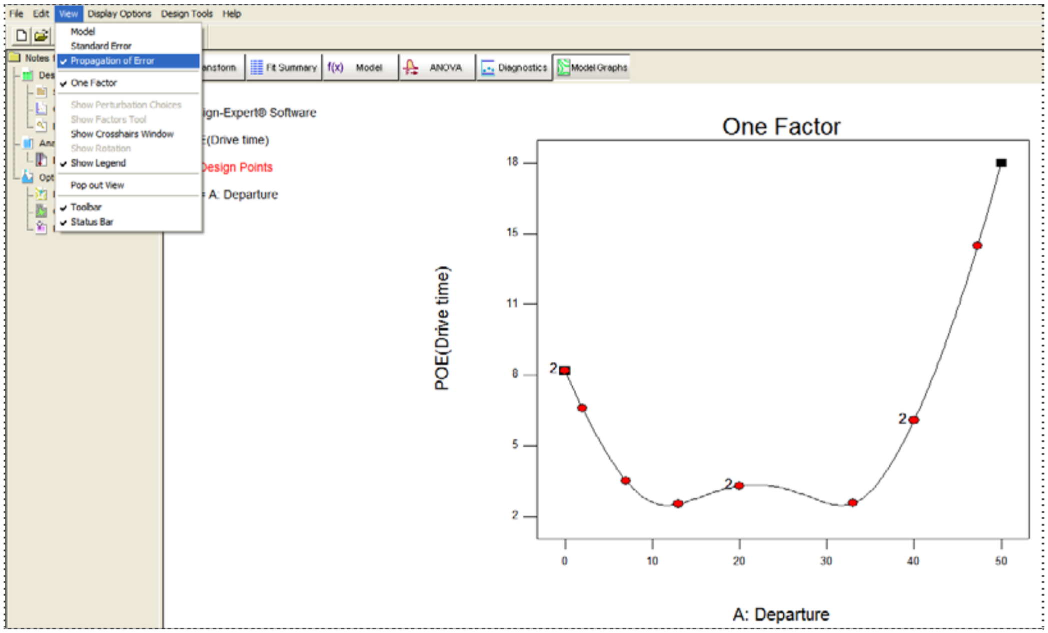
Editing info for the input factor

For **Std Dev** enter **5**.



Entering standard deviation for factor

Press **OK** and then go back to the **Analysis** branch, click the **Drive time** node and return to ***Model Graphs***. Then from the **View** menu select **Propagation of Error**.



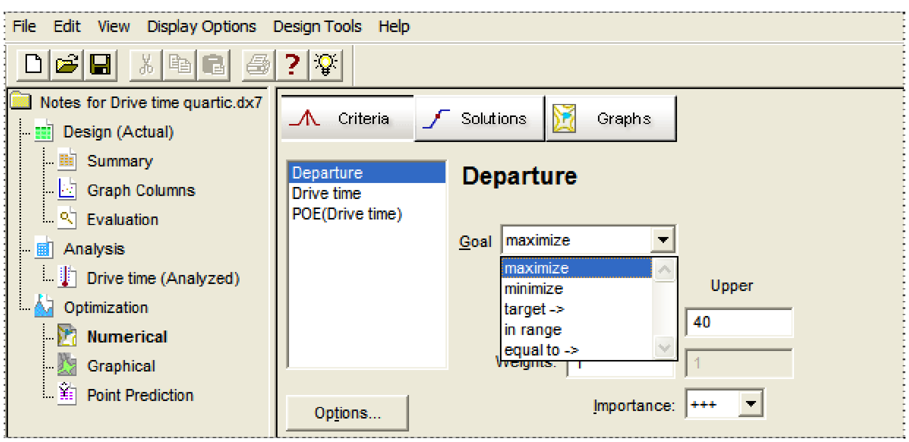
Plot for POE

Notice that POE is minimized at two times for departure, which correspond with flats on the wavy response plot you looked at earlier.

Multiple Response Optimization

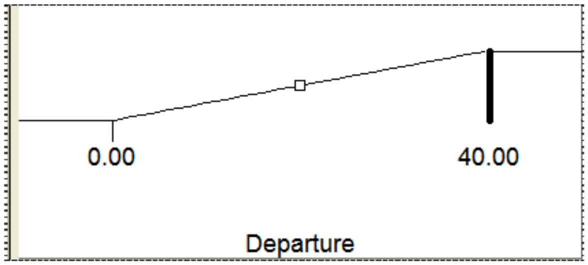
Ideally, Mark would like to leave as late as possible (to get more sleep every morning!) while minimizing his drive time but making it the least variable. These goals can be established in Design-Expert software so it can look for the most desirable outcomes.

Under the **Optimization** branch choose the **Numerical** node. For the Departure, which comes up by default, click the Goal and select maximize.



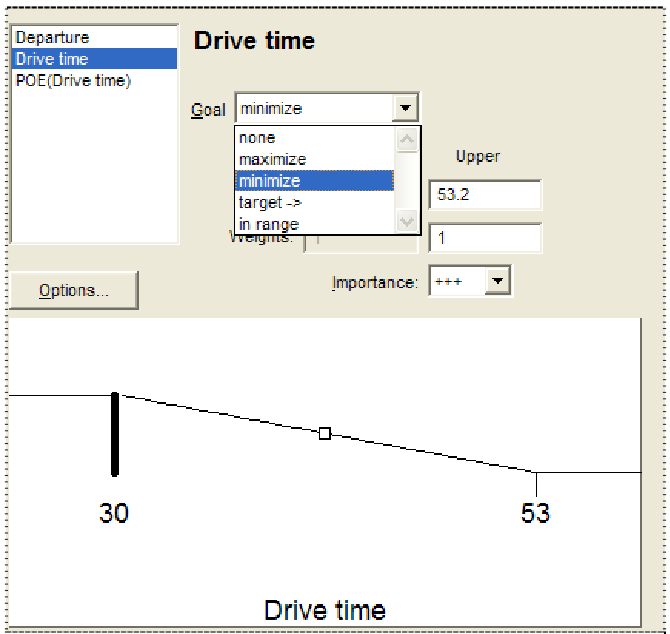
Setting goal for departure

The program pictures this goal as an upward ramp (/) to indicate that the higher this variable goes the more desirable it becomes.



Desirability ramp for departure – later the better (maximize)

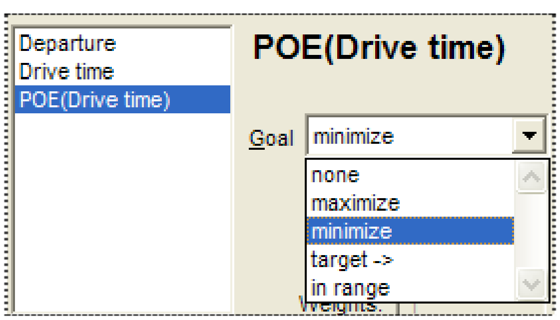
Next, click on the response for **Drive time** and for its **Goal** select **minimize**.



Drive time minimized

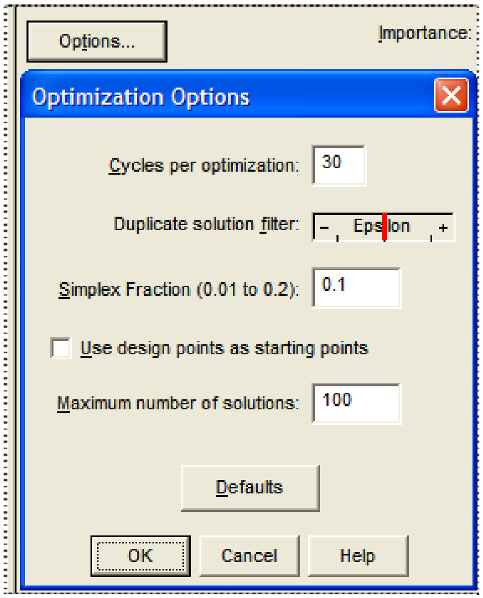
Notice the ramp now goes downward (\) to show that for this variable lesser is better, that is, more desirable.

Lastly, to reduce variation in drive time caused by deviation in departure, click **POE** and set its **Goal** to **minimize**.



Minimizing POE

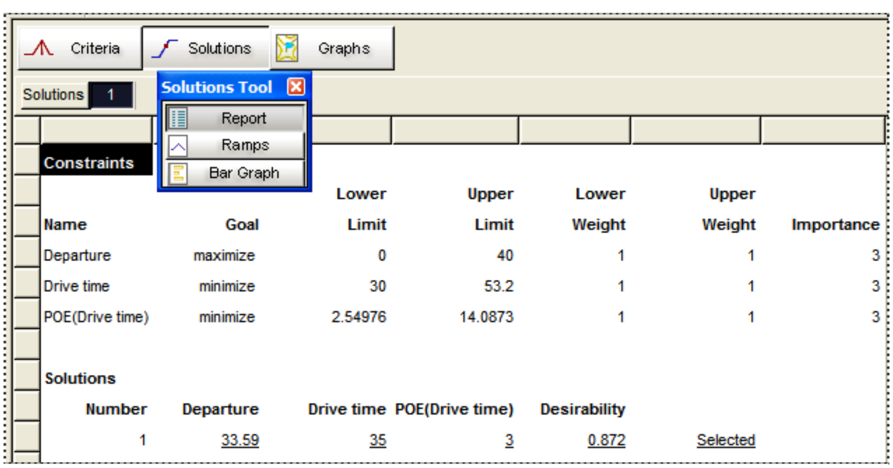
Before pressing ahead, click the **Options** button.



Options for numeric optimization

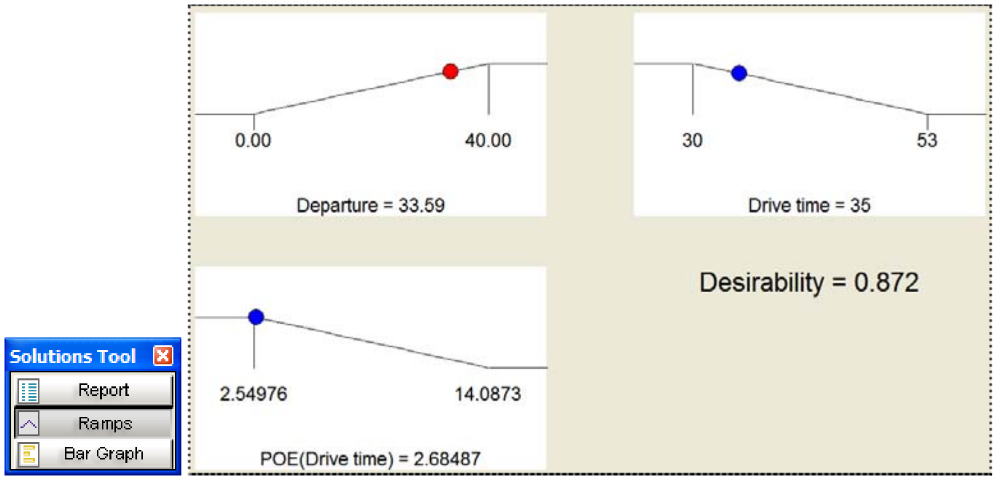
The settings here will affect the hill-climbing algorithm that Design-Expert uses to find the most desirable combination of variables. For details, check Help. Click **OK** to accept the defaults.

Press the ***Solutions*** button to see what Design-Expert recommends for the most desirable departure. The program now chooses a departure time at random and climbs up the desirability response surface. It repeats this process over and over (100 times by default), but in this case, the same point (within a value "epsilon" for the duplicate solution filter – see Optimization Options above) is found every time – a departure around 33 minutes beyond the earliest start acceptable by Mark for his morning commute. (Your result may vary somewhat due to the random starting points of the hill-climbing algorithm.)



Most desirable solution (your result may vary somewhat)

For a quick feel for optimal factor settings and the most desirable results, on the **Solutions Tool** click **Ramps**.



Ramps view of most desirable solution

Now Mark knows when its best to leave for work while simultaneously maximizing the departure (and gaining more 'shut-eye'), minimizing his drive time, and minimizing propagation of error. Now the only thing that could possibly go wrong would be if all the other commuters learn how to use RSM and make use of Design-Expert. Mark hopes that none of you who are reading this tutorial live in his suburban neighborhood and work downtown.