

Split-Plot General Factorial Tutorial

Introduction

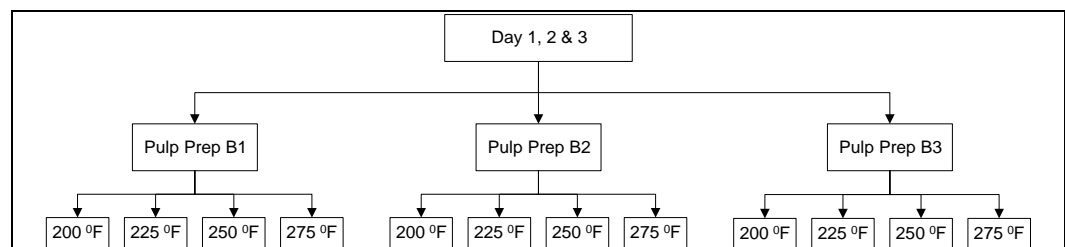
In some randomized block designs you must restrict the randomization. Otherwise it wouldn't be practical to perform the experiments. For example, temperature cannot be easily changed in many applications. The solution may be a "split-plot" design, which originated in the field of agriculture. Experimenters divided large areas of land, called "whole plots," into smaller "subplots" that could be treated separately. For example, they planted several crop varieties - one each per whole plot, and then applied different fertilizers on each subplot.

The analysis of a split plot design is tricky, even for statisticians. It can be done with Design-Expert® software by properly designating effects in specific ways for subsequent analysis of variance. Proceed if you dare!

To illustrate how Design-Expert can be manipulated to do a split-plot, let's do an example from Montgomery's *Design and Analysis of Experiments*. A manufacturer wants to determine the effect of three methods of pulp preparation and four levels of cooking temperature on the tensile strength of paper. These twelve combinations can be done in one day.

To increase statistical power, the manufacturer decides to do three replicates of this general factorial design over three days, which produces a total of 36 runs. However, it will not be feasible to perform the experiment in a random fashion. Instead, on each of the three days, the experimenter produces a batch of pulp and divides it into four samples to be cooked at four different temperatures. This process is repeated for pulp prepared by the two other methods.

A flowchart of the experiment is shown below.



Flowchart of split plot experiment on papermaking process

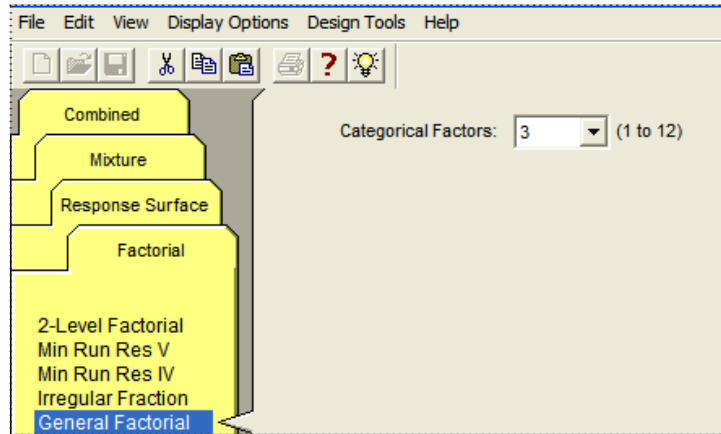
In statistical terms, the split plot experiment can be structured as:

- Whole plots for the three batches of pulp
- Sub-plots for the four samples cooked at four different temperatures.

You will set up this design as a general factorial. It will then need to be modified somewhat so it comes out in the right order. Then you will perform an analysis, which really gets tricky due to the restrictions in randomization.

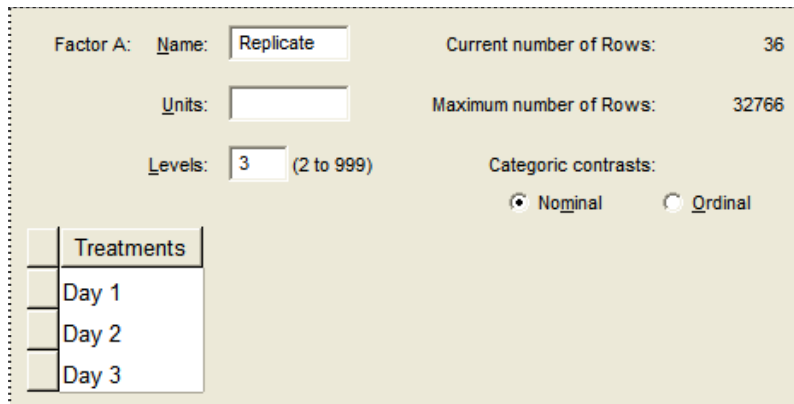
Design the Experiment

Let's build the design. This is done via the usual procedure for general factorials, but with days explicitly accounted for as a factor. [Click the blank-sheet icon](#) on the toolbar (or choose File, New Design). Then from the default **Factorial** tab, click on **General Factorial**. Enter **3** as the number of factors.



Setting up the general factorial design

Click **Continue** and enter **Replicate** for the factor name, **3** for the number of levels, and **Day 1**, **Day 2** and **Day 3** for the treatment names. Your screen should now look like the illustration below.



Entering first factor – days

Click **Continue** to move on to the next factor. Then enter **Pulp Prep** for the factor name, and **3** for the number of levels. Change the treatment names to **B1**, **B2** and **B3**.

Factor B: Name: Pulp Prep Current number of Rows: 36

Units: Maximum number of Rows: 32766

Levels: 3 (2 to 999) Categorical contrasts:

Nominal Ordinal

Treatments
B1
B2
B3

Second factor: Pulp prep

Then **Continue** to the final (third) factor and name it **Temp** with **deg F** for units, **4** for the number of levels and **200, 225, 250** and **275** for the treatments. Change the **Categorical contrasts** to **Ordinal**, thus recognizing its numeric nature.

Factor C: Name: Temp Current number of Rows: 36

Units: deg F Maximum number of Rows: 32766

Levels: 4 (2 to 999) Categorical contrasts:

Nominal Ordinal

Treatments
200
225
250
275

Last factor changed to “ordinal”

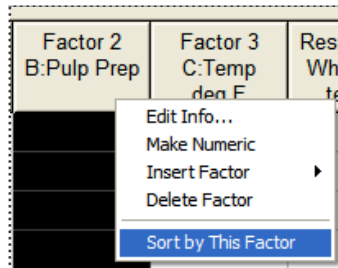
Press **Continue**, leave “1” for replicates as the default to produce 36 runs (=3x3x4 levels from the three factors), and click on the **Continue** button. Although only one response was measured in this experiment, it will be broken down four ways (to be explained later), so select **4** and enter the names shown below. If you are adept at copying and pasting via Edit or shortcut keys, Ctrl-C and Ctrl-V, respectively, use these tools to save on keystrokes for entering units as **tensile**.

Responses: 4 (1 to 5)

	Name	Units
	Whole plot	tensile
	Sub plot	tensile
	Interaction	tensile
	All effects	tensile

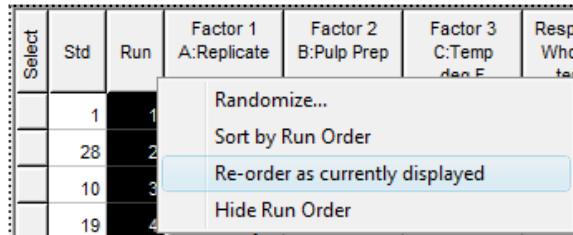
Entering response name and units

Don't bother filling in any of the other fields that relate to calculating power for the design – simply press **Continue** to complete the design-building wizard. Design-Expert now displays the design in completely random order. Obviously this cannot be performed as listed: Day 1 must precede day 2, all temperatures must be run on one method of pulp preparation before starting on the next, and so forth. To modify the order, first right click on the **Pulp Prep** (Factor 2) column heading and choose **Sort by This Factor**.



Sorting by Pulp Prep method


Next, right click on the **Replicate** (Factor 1) column and select **Sort by This Factor**. Now you see subplot treatment (C - cooking temperature) randomized within the whole plot treatment (B - pulp preparation). To retain this order, right click on the **Run** column header and select Re-order as currently displayed.

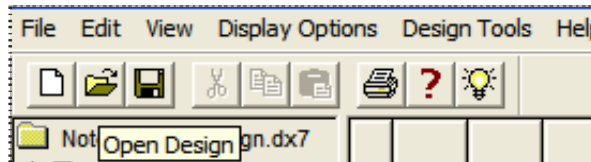


After sorting by Replicate select Run re-order as currently displayed

At this stage the design would normally be printed as a run-sheet for the actual experiment, with resulting responses of tensile strength recorded.

Analyze the Results

To avoid the trouble of typing in data, simply read in the responses via the  icon or via File, Open Design from the main menu.



Opening the design file to get the response data

Select the file named **Paper.dx7**. You should now see the data depicted in the table below, but with the tensile strength results duplicated four times in preparation for the various ways that it will be analyzed.

Run	A: Rep	B: Pulp Prep	C: Temp deg F	Tensile Strength
1	Day 1	B1	275	36
2	Day 1	B1	250	37
3	Day 1	B1	200	30
4	Day 1	B1	225	35
5	Day 1	B2	275	42
6	Day 1	B2	250	38
7	Day 1	B2	200	34
8	Day 1	B2	225	41
9	Day 1	B3	225	26
10	Day 1	B3	275	36
11	Day 1	B3	250	33
12	Day 1	B3	200	29
13	Day 2	B1	225	32
14	Day 2	B1	250	40
15	Day 2	B1	200	28
16	Day 2	B1	275	41
17	Day 2	B2	275	40
18	Day 2	B2	225	36
19	Day 2	B2	250	42
20	Day 2	B2	200	31
21	Day 2	B3	275	40
22	Day 2	B3	225	30
23	Day 2	B3	250	32
24	Day 2	B3	200	31
25	Day 3	B1	250	41
26	Day 3	B1	225	37
27	Day 3	B1	275	40
28	Day 3	B1	200	31
29	Day 3	B2	275	44
30	Day 3	B2	225	40
31	Day 3	B2	200	35
32	Day 3	B2	250	39
33	Day 3	B3	275	45
34	Day 3	B3	250	39
35	Day 3	B3	225	34
36	Day 3	B3	200	32

Response data for tensile strength of paper

The analysis of a split plot design in Design-Expert must be done somewhat manually. You need to create separate ANOVA's for the whole plot treatment (pulp prep), the subplot treatment (temperature) and the interaction between whole plot and subplot individually to get the correct statistical tests for each. Then you fit the full model in order to get meaningful diagnostics and model graphs (but you ignore the full model

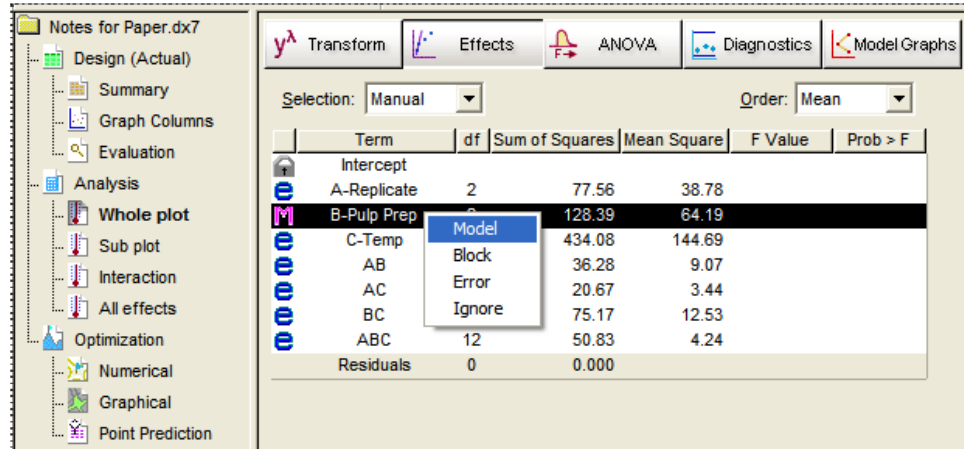
ANOVA!). To keep these various analyses straight, the tensile strength results have been copied four times over.

To analyze the whole plot treatment (B - pulp preparation) click on the Analysis node labeled **Whole plot** which can be found in the tree structure along the left of the main window. Then click on the **Effects** button displayed in the progressive toolbar.

Design-Expert software supports four alternative effect designations:

- Model (“M”)
- Block (“b”)
- Error (“e”)
- Ignore (“X”)

The whole plot treatment (B) should be tested against the Replicate by Pulp Preparation interaction, i.e., the AB interaction. In the effects list right click on factor **B** and choose **Model** (or you could simply press M). Another way to designate a term for model is to double-click it. Try this on B if you like. It toggles back to error. Double click the term again to put it in the model.



Right-click menu for designating effects

In order to include a block correction in the model, right click on factor **A** and choose **Block** (mark with “b”). Right click on **C** and **Ignore** it (mark with “X”). Do the same for AC, BC and ABC (suggestion: try clicking on AC and shift-clicking on ABC to highlight all three effects, and then do one right-click to designate this block of effects to be ignored). Leave AB at its default designation as Error (“e”). Your screen should now match the illustration below.

Term	df	Sum of Squares	Mean Square	F Value	Prob > F
Intercept					
A-Replicate	2	77.56	38.78		
B-Pulp Prep	2	128.39	64.19		
C-Temp	3	434.08	144.69		
AB	4	36.28	9.07		
AC	6	20.67	3.44		
BC	6	75.17	12.53		
ABC	12	50.83	4.24		
Model	0	0.000			

Effects screen after designating effects for analyzing whole plot treatments

To recap, the designation of terms can be accomplished in numerous ways. As noted earlier they can be toggled from model to error (and back) by a double-click with your mouse. You've seen the right-click menu approach to designating terms, which provides a broader selection of option. Either click on the one you want, or enter its first letter. (No need to press the Alt key!) Finally, if you want to ignore a term, you can press the delete key. Go ahead and try all of the above, but be sure to return your selections to those shown above before proceeding.

Click on the **ANOVA** button. You can see that the whole plot treatment, pulp preparation (factor B), is significant. Ignore the remainder of the output for now, because this is only part of the picture.

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Block	77.56	2	38.78			
Model	128.39	2	64.19	7.08	0.0485	significant
B-Pulp Prep	128.39	2	64.19	7.08	0.0485	
Residual	36.28	4	9.07			
Cor Total	242.22	8				

ANOVA for whole plot (main effect of B – the pulp preparation)

Do not look at the Diagnostics or the Model Graphs because the models are incomplete at this point. However, to preserve your ANOVA on the whole plot treatment (B – pulp preparation), select **File, Save As** and modify the name to **Paper-a.dx7** (or anything else you'd like that will leave the original tutorial file as-is).

To analyze the subplot treatment (C - temperature), click the **Sub plot** node for analysis and press the **Effects** button. The subplot treatment (C) should be tested against the Replicate by Temperature interaction, i.e., the AC interaction. Right click on factor **B** and choose **Block**, choose **C** for the **Model**, **AC** for the **Error**. All other terms (the interactions **AB**, **BC** and **ABC**) should be set to **Ignore**. Your screen should now match the illustration shown below.

Term	df	Sum of Squares	Mean Square
Intercept			
A-Replicate	2	77.56	38.78
B-Pulp Prep	2	128.39	64.19
C-Temp	3	434.08	144.69
AB	4	36.28	9.07
AC	6	20.67	3.44
BC	6	75.17	12.53
ABC	12	50.83	4.24
Residuals	0	0.000	

Designating effects for analyzing subplot treatments

Click on the **ANOVA** button. You can see that the subplot treatment, cook temperature (factor C), is significant. Ignore the remainder of the output.

ANOVA for selected factorial model

Block term includes A, B
Error term includes AC

Ignored terms are AB BC ABC

SS: 162.278 df: 22

Analysis of variance table [Classical sum of squares - Type II]

Source	Sum of Squares	df	Mean Square	F Value	p-value	
					Prob > F	
Block	205.94	4	51.49			
Model	434.08	3	144.69	42.01	0.0002	significant
C-Temp	434.08	3	144.69	42.01	0.0002	
Residual	20.67	6	3.44			
Cor Total	660.69	13				

ANOVA for sub-plot

Do not look at the Diagnostics or the Model Graphs because the models are incomplete at this point. However, to preserve your ANOVA, select **File, Save** or simply click the save icon .

To analyze the whole plot by subplot treatment interaction, click on the **Interaction** node. Then press the **Effects** button. The whole plot by subplot interaction (BC) should be tested against the ABC interaction. Right click on factor **C** and choose **Block**. Choose **BC** for the **Model** and set the **AC** term to **Ignore**. Leave the **ABC** at its default of **e** (for error). Your screen should now match the following illustration.

Term	df	Sum of Squares	Mean Square
Intercept			
A-Replicate	2	77.56	38.78
B-Pulp Prep	2	128.39	64.19
C-Temp	3	434.08	144.69
AB	4	36.28	9.07
AC	6	20.67	3.44
BC	6	75.17	12.53
ABC	12	50.83	4.24
Residuals	0	0.000	

Designating effects for whole plot by sub-plot interaction

Click on the **ANOVA** button.

Use your mouse to right click on individual cells for definitions.

Response 3 Interaction

ANOVA for selected factorial model

Block term includes A, B, C

Error term includes ABC

Ignored terms are AB AC

SS: 56.9444 df: 10

Analysis of variance table [Classical sum of squares - Type II]

Source	Sum of Squares	df	Mean Square	F Value	p-value	Prob > F
Block	640.03	7	91.43			
Model	75.17	6	12.53	2.96	0.0520	not significant
BC	75.17	6	12.53	2.96	0.0520	
Residual	50.83	12	4.24			
Cor Total	766.03	25				

ANOVA for whole plot by sub-plot interaction

The model in annotated view is labeled as “insignificant.” However, this is arbitrarily based on Design-Expert program’s default for a significance threshold of 0.05 (modifiable via Edit, Preferences, Math tab). Recall that earlier the effect of B (the whole plot treatment) was deemed significant at a p-value of 0.0485. The Prob > F for

BC is only slightly higher at 0.052. Therefore, the experimenters decided to ignore the advice from the software, and consider BC to be significant.

Do not look at the Diagnostics or the Model Graphs because the models are incomplete at this point.

Before moving on to the final phase of the analysis, we want to provide some justification for the statistical analysis presented up to this point.

- In a blocked design the block-by-treatment interactions are used to estimate error.
- In a completely randomized blocked design all the block-by-treatment interactions are combined into a single estimate of error.
- Due to the restrictions on randomization in a split plot design, specific block-by-treatment interactions must be designated to estimate error for the particular treatment being tested. This is facilitated in this case by entering the blocks (day-by-day) as factor A.

Here's a recap of how you performed the analyses of variance (ANOVAs) for this experiment:

1. The whole plot treatment (B-pulp preparation) against the “block by pulp” (AB) interaction.
2. The sub-plot treatment (C-temperature) against the “block by temperature” (AC) interaction.
3. The pulp by temperature (BC) interaction against “block by pulp by temperature” (ABC) interaction.

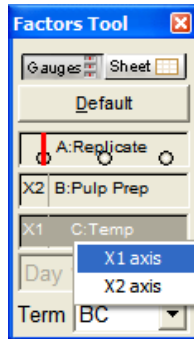
To get meaningful diagnostics and model graphs, you now must fit the full model by clicking the **All effects** node. Click on **Effects** button and right-click A to Block. Then choose **B, C** and **BC** for the **Model**. Leave **AB, AC** and **ABC** at their defaults of **e** for error. Your screen should now match that shown below.

Term	df	Sum of Squares	Mean Square
Intercept	1	77.56	77.56
A-Replicate	2	128.39	64.19
B-Pulp Prep	2	434.08	217.04
C-Temp	3	144.69	48.23
AB	4	36.28	9.07
AC	6	20.67	3.44
BC	6	75.17	12.53
ABC	12	50.83	4.24
Residuals	0	0.000	0.000

Fitting the full model for purposes of doing diagnostics and model graphs

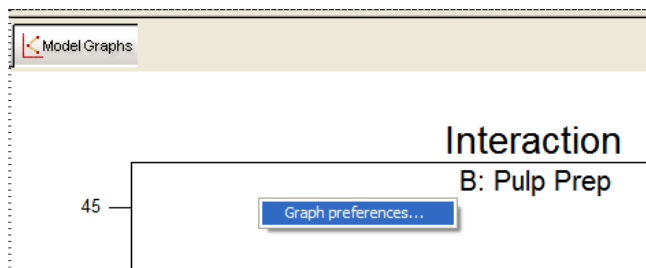
Skip over the ANOVA (been there and done that) and click directly on the **Diagnostics** button. Examine all the graphs available via the Diagnostics tool. You

should see nothing abnormal, so press ahead to **Model Graphs**. On the **Factors Tool** right click over the **C: Temperature** bar and change it to the **X1 axis**.



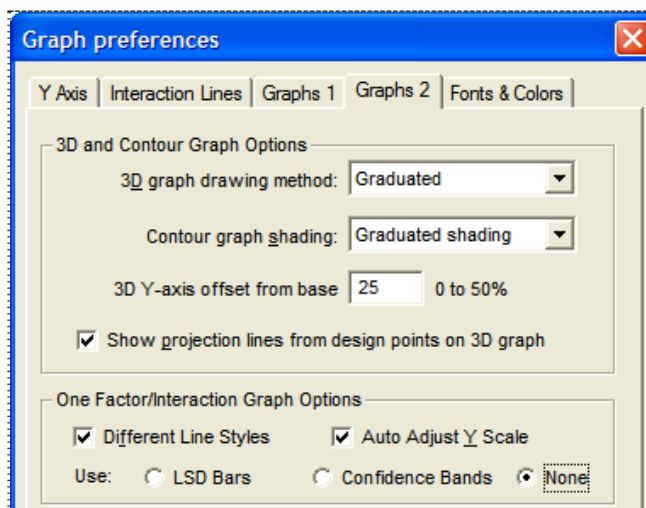
Changing axes

You now see an interaction plot of the continuous factor temperature at three discrete levels of pulp preparation. The LSD (least significant difference) bars that come up by default are incorrect, so they need to be turned off via right-clicking over the plot and selecting **Graph preferences**.



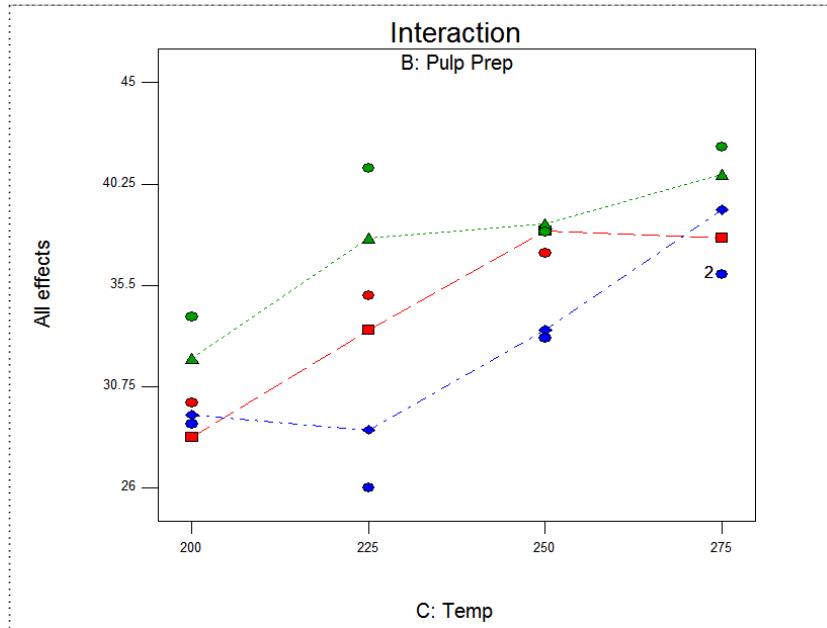
Effects graph – Graph preferences

Then click **Graphs 2** and press the option for **None**.



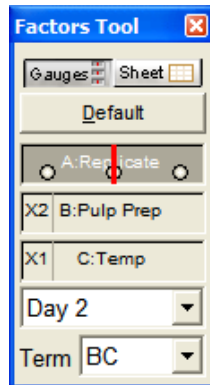
Turning off the LSD bars

Now your interaction plot should look like that shown below.



Effects graph – interaction of temperature by pulp preparation on Day 1

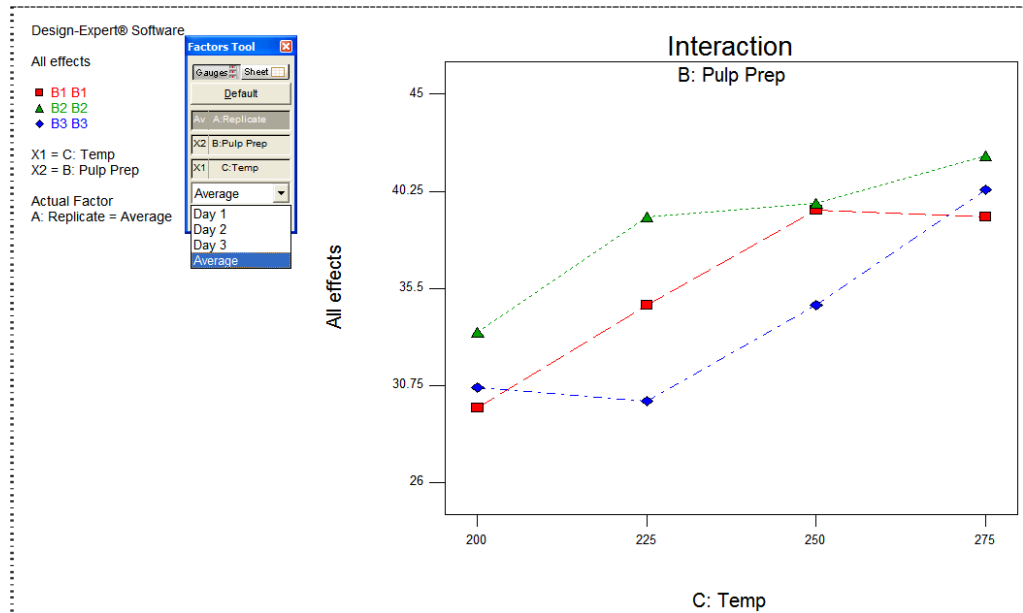
By default, the graph comes up for Day 1 (lowest level of factor A). On the **Factors Tool** press the second button on **Replicate** to see the predicted results for Day 2.



Setting Replicate to Day 2


Notice how the graph shifts very slightly. Click on any point to get the identification. The round symbols represent actual values, which are displayed to the left of the plot when you click the point. The other symbols (triangle, square and diamond in varying colors) show the predicted means. Click any of these points to get the value for tensile strength.

Now press the third button to see the predicted results for Day 3. However, since time cannot be controlled, it might be best to show the BC interaction at an average of the Days. As shown below, click on the list arrow by the **Day** field and select **Average**.



Setting replicates to average level

From this graph we can see pulp preparation B2 will maximize tensile strength. Evidently it will be most robust to temperature changes, especially in the temperature range of 225 to 275 degrees F. This relationship holds true regardless of which day (replicate) you select (the buttons on the tool palette).

To preserve the modeling used to produce the plots, select **File, Save** or click the save icon .

This is the end of the story, but as a postscript, you may wonder if it was worth all the bother to properly account for the split-plot nature of this experiment. Click back to the ANOVA for this “All effects” analysis and then back to the ones for “Whole”, “Sub plot” and “Interaction.” The differences are relatively subtle. In other words, in this particular case, an experimenter who unwittingly ignored the restrictions in randomization caused by the split plot may have arrived at the same general conclusions shown here. In any case, it is always wise to assess the practical importance of the observed effects and, if warranted, follow up by doing confirmation runs to ensure that predictions from the model will be relatively accurate.