Multifactor RSM Tutorial (Adding categorical factors)

Combining categorical with numeric factors

Procter and Gamble (P&G) engineers on a new sealing process were concerned about how maximum peel strength would be affected by changing suppliers of the packaging material (Brenneman, William A., and William R. Myers, "Robust Parameter Design with Categorical Noise Variables," *Journal of Quality Technology* 35, no. 4 (October 2003): 335-341). They set up an RSM design to vary several key factors on the sealing machine, including the supplier:

A. Temperature, 193 to 230 degrees C.

B. Pressure, 2.2 to 3.2 Bar.

C. Speed, 32 to 50 cpm (cycles per minute).

D. Supplier: S1, S2, S3.

Due to limitations on time and other resources, a maximum of 37 runs could be performed. Therefore, simply doing a standard CCD or BBD for each of the three suppliers would not do–these design choices produce far too many runs (60 and 51, respectively). Instead, the P&G engineers made use of a D-optimal design.

The data below come from a simulation that's loosely based on the predictive model reported in the cited article. (Some liberties were taken to make the outcome more interesting.) Assume that the maximum peel strength will ideally hit a target of 4.5 pound-force (lbf). However, it must exceed 3 lbf to prevent leaking and not go above 6 lbf because the package becomes too difficult to open.

The tutorial explains how this design was constructed via D-optimal criteria with added points for lack of fit testing and pure-error estimation. Notice that we made use of nearly all the budgeted runs. Also, aided by a feature in the software that forces balance, we made sure that each supplier got an equal number of design runs.

With guidance from the tutorial you will analyze the responses and then find desirable solutions for the following objectives–listed according to relative acceptability for all concerned.

1. Can the process be adjusted to hit the maximum peel strength target of 4.5 lbf for any one of the suppliers?

2. The purchase agent intended to break down the supply in this manner: 50% to S1, 25% to S2 and 25% to S3. Can you find a process setup that will work for all suppliers? If not, a two-supplier option might be satisfactory, provided S1 is one of them. In other words, assuming it will not be robust for all three suppliers, can the process be set up in such a way that either S1-S2 or S1-S3 meets specifications?

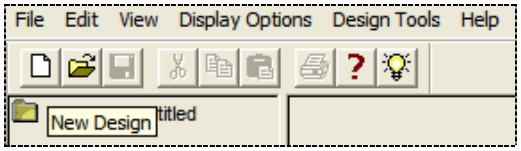
3. If it is not possible to achieve a common setup for even two of the three suppliers, perhaps the upper limit of 6 could be raised on the maximum peel strength specification. (Assume that customers are willing to use a scissors or, better yet, the package designers can add a notch in the plastic for easier opening.) Will this open up a window of operability for multiple suppliers at one set of process conditions?

Std	A: Temp (deg C)	B: Pressure (Bar)	C: Speed (cpm)	D: Supplier	Peel Strength (lbf)
1	193	2.2	32	S1	4.6
2	230	2.2	32	S1	10.0
3	193	3.2	32	S1	7.2
4	193	3.2	32	S1	6.6
5	230	3.2	32	S1	5.5
6	202.25	2.7	41	S1	8.2
7	230	2.7	41	S1	8.5
8	193	2.2	50	S1	6.7
9	230	2.2	50	S1	11.0
10	230	2.2	50	S1	11.0
11	230	3.2	50	S1	7.8
12	230	3.2	50	S1	7.1
13	230	2.2	32	S2	5.1
14	230	2.2	32	S2	6.7
15	211.5	2.7	32	S2	2.9
16	230	3.2	32	S2	1.7
17	211.5	2.2	41	S2	6.3
18	193	2.7	41	S2	2.0
19	220.75	2.95	41	S2	3.7
20	193	2.2	50	S2	2.4
21	230	2.7	50	S2	4.0
22	230	2.7	50	S2	4.1
23	193	3.2	50	S2	4.8
24	193	3.2	50	S2	5.1
25	193	2.2	32	S3	6.7
26	193	2.2	32	S3	6.7
27	230	3.2	32	S3	8.0
28	230	3.2	32	S3	7.1
29	220.75	2.45	36.5	S3	9.2
30	202.25	2.7	36.5	S3	8.1
31	211.5	3.2	41	S3	7.2
32	230	2.2	50	S3	9.5
33	230	2.2	50	S3	9.8
34	211.5	2.7	50	S3	6.2
35	193	3.2	50	S3	5.9
36	230	3.2	50	S3	4.8

Data from package sealing experiment

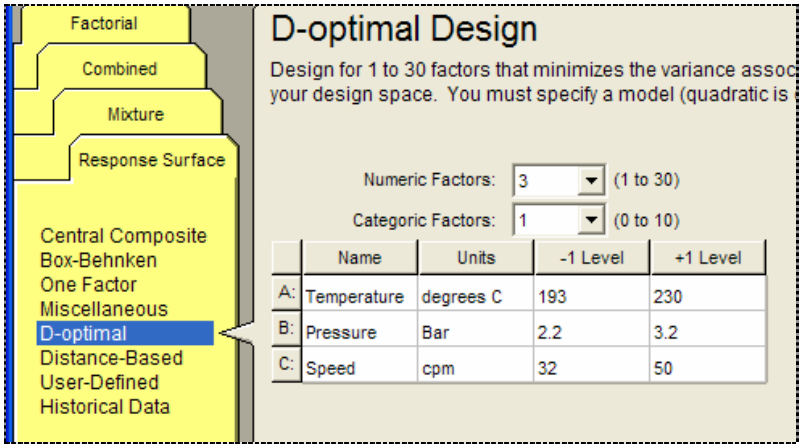
Design the Experiment

Start Design-Expert® software and initiate the design process by clicking the blank-sheet icon 🗋 on the left of the toolbar or select File, New Design.



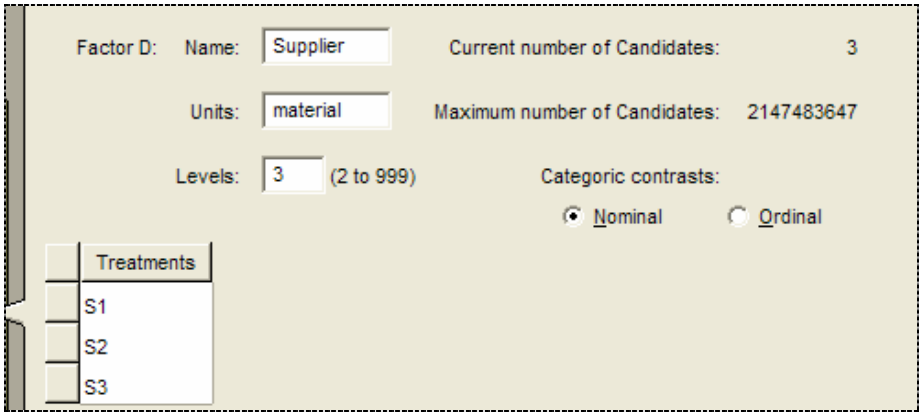
Main menu and tool bar

Click on the **Response Surface** folder tab, select **D-optimal** for the design. For **Numeric Factors** enter **3** and for **Categoric Factors** select **1**. Then enter the **Name**, **Units**, **-1 Level** and **+1 Level** for the numeric factors A, B and C as shown below.



Entering factor levels for d-optimal design

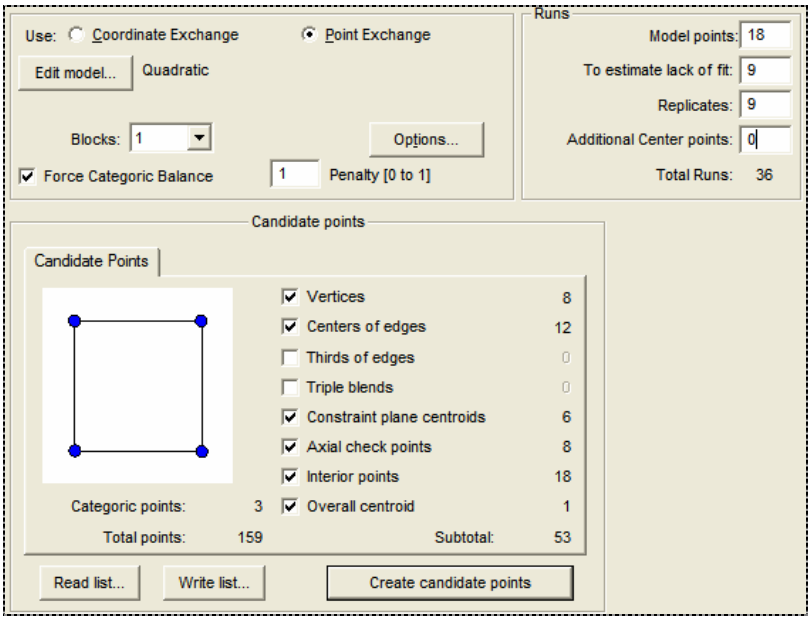
Press **Continue** and, as shown below, enter the **Name**, **Units** and number of **Levels** for the categoric factor. Then mouse down to the **Treatments** fields and identify them as indicated on the screen shot.



Entering the categoric factor

The default selection of “Nominal” contrasts applies here because the treatments are discrete names. If you entered treatments that followed an order, such as “1”, “2” and “3”, it would be best to set these up with “Ordinal” contrasts. This would affect construction of the model and the layout of ANOVA.

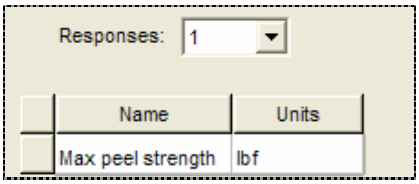
Press **Continue** to specify the d-optimal design. Select the option for **Point Exchange**. The defaults for runs add up to a total of 28, which is not divisible by 3 – desirable for splitting out evenly to the three suppliers. Bulk up the design to a total of 36 runs (thus providing 12 to each supplier) by increasing those **To estimate lack of fit** to **9** and the **Replicates** also to **9**. Click on the **Force Categoric Balance**. Then press **Create candidate points**. Your screen should now match the one below.



D-optimal specifications modified

The program now identifies 53 candidate points for each of the 3 suppliers for a total of 159, from which it will select d-optimally the 18 points needed at a minimum for the quadratic model. Then it will add 9 more runs with uniquely different combinations of factors for testing lack of fit. Finally, 9 of the 27 points already identified will be replicated. For details on all this and the force-balance feature for categoric factors, refer to program Help.

Press **Continue** and enter the response Name and Units as shown below.

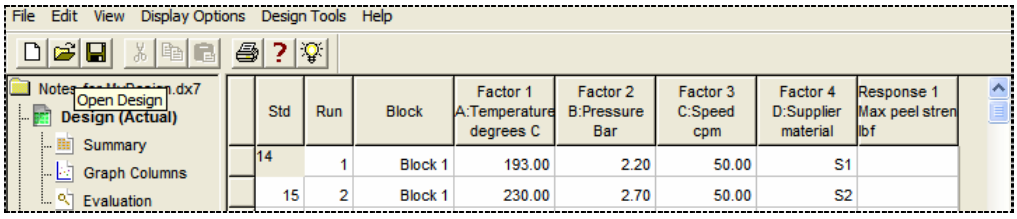


Response names and units

Press **Continue** again to bring up the design template.

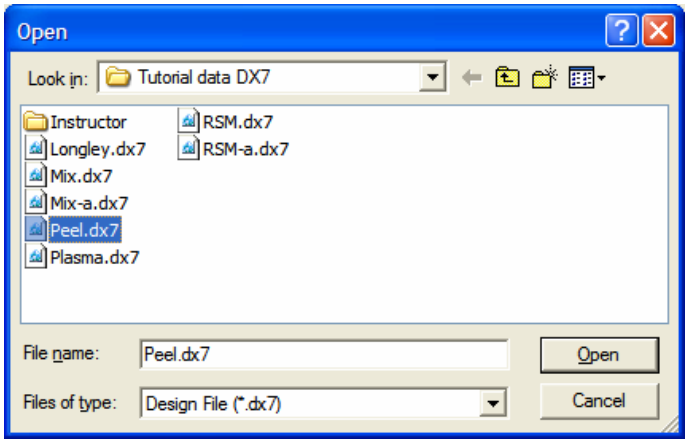
Analyze the Response

Your layout may match what's shown in the original data table, but there will be many possible point selections that would be equally good. To avoid the possibility of a mismatch, but more importantly, to save you the time of entering responses, click the open design icon (the one that looks like a folder that's shown below) or select File, Open Design.



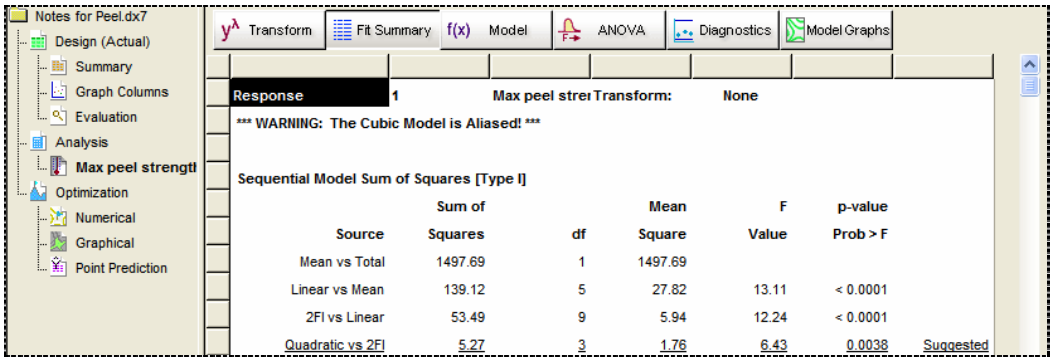
Opening a design

Then find the file named **Peel.dx7** and Open it.



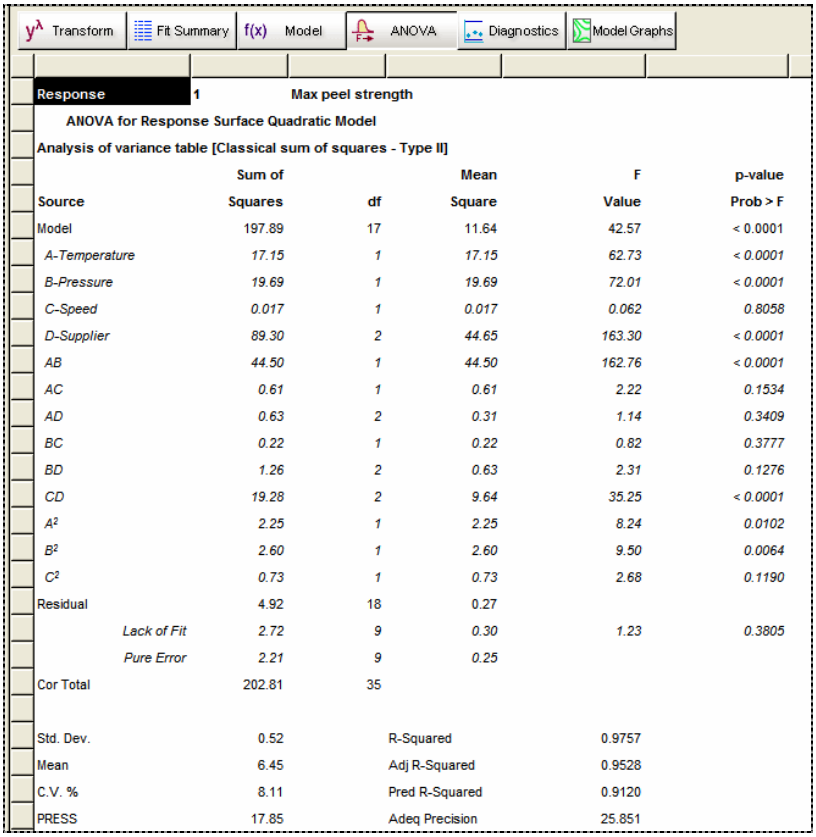
Opening the file previously created for this case

Under the **Analysis** branch click the response for **Max peel strength**. Then press the ***Fit Summary*** button.



Fit summary report

Notice that the design-for model, quadratic, is suggested. Scroll down and see that it exhibits no significant lack of fit. Press ahead to ***ANOVA*** with this chosen model.



Response 1 Max peel strength

ANOVA for Response Surface Quadratic Model

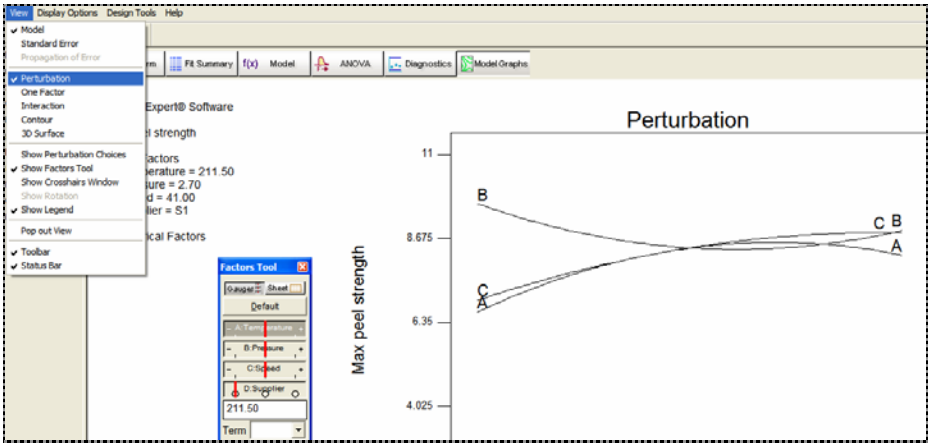
Analysis of variance table [Classical sum of squares - Type II]

Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F
Model	197.89	17	11.64	42.57	< 0.0001
A-Temperature	*17.15*	*1*	*17.15*	*62.73*	*< 0.0001*
B-Pressure	*19.69*	*1*	*19.69*	*72.01*	*< 0.0001*
C-Speed	*0.017*	*1*	*0.017*	*0.062*	*0.8058*
D-Supplier	*89.30*	*2*	*44.65*	*163.30*	*< 0.0001*
AB	*44.50*	*1*	*44.50*	*162.76*	*< 0.0001*
AC	*0.61*	*1*	*0.61*	*2.22*	*0.1534*
AD	*0.63*	*2*	*0.31*	*1.14*	*0.3409*
BC	*0.22*	*1*	*0.22*	*0.82*	*0.3777*
BD	*1.26*	*2*	*0.63*	*2.31*	*0.1276*
CD	*19.28*	*2*	*9.64*	*35.25*	*< 0.0001*
A^2	*2.25*	*1*	*2.25*	*8.24*	*0.0102*
B^2	*2.60*	*1*	*2.60*	*9.50*	*0.0064*
C^2	*0.73*	*1*	*0.73*	*2.68*	*0.1190*
Residual	4.92	18	0.27		
Lack of Fit	*2.72*	*9*	*0.30*	*1.23*	*0.3805*
Pure Error	*2.21*	*9*	*0.25*		
Cor Total	202.81	35			

Std. Dev.	0.52	R-Squared	0.9757
Mean	6.45	Adj R-Squared	0.9528
C.V. %	8.11	Pred R-Squared	0.9120
PRESS	17.85	Adeq Precision	25.851

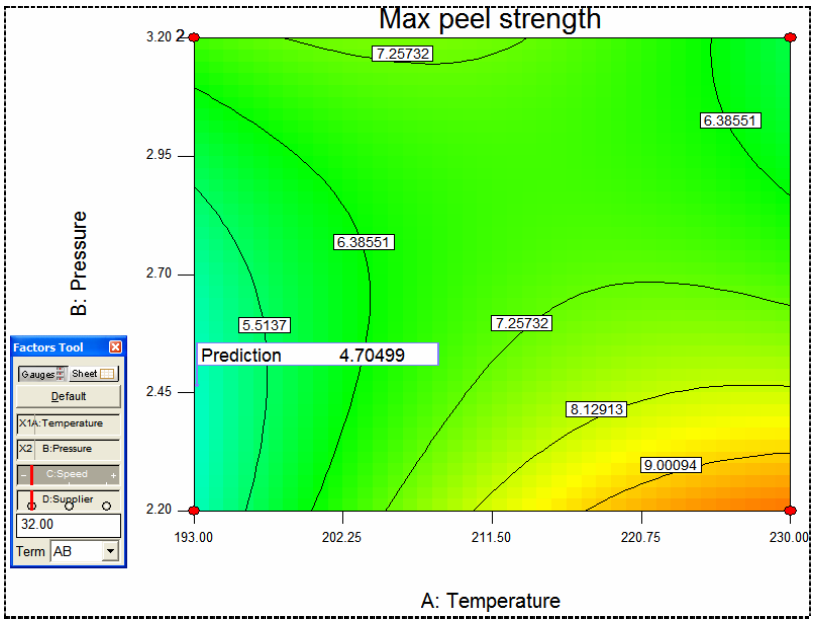
ANOVA for quadratic model

This looks very good. Some terms come out insignificant but carry them along to ***Diagnostics*** (look these over – but they shouldn't alarm you) and then ***Model Graphs***. This is where things get interesting, because several suppliers and the purchase agent are all anxious to see how they've done! Select **View**, **Perturbation** to get an overview plot before generating contours and 3D surfaces.



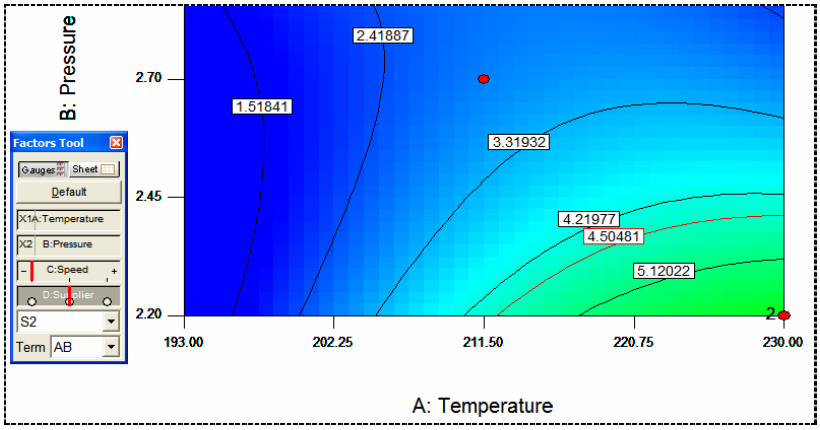
Perturbation plot

Keep in mind that 4.5 lbf is the target, so this supplier (S1 – chosen by default) is coming out high. Lowering factor C will help. Go back to **View**, **Contour** and on the **Factors Tool** click on **C:Speed** and slide it left. At the lowest response levels, the cool blue-green on the graduated color shading done by default on contour plots, see what's predicted via a right-click and Add flag.



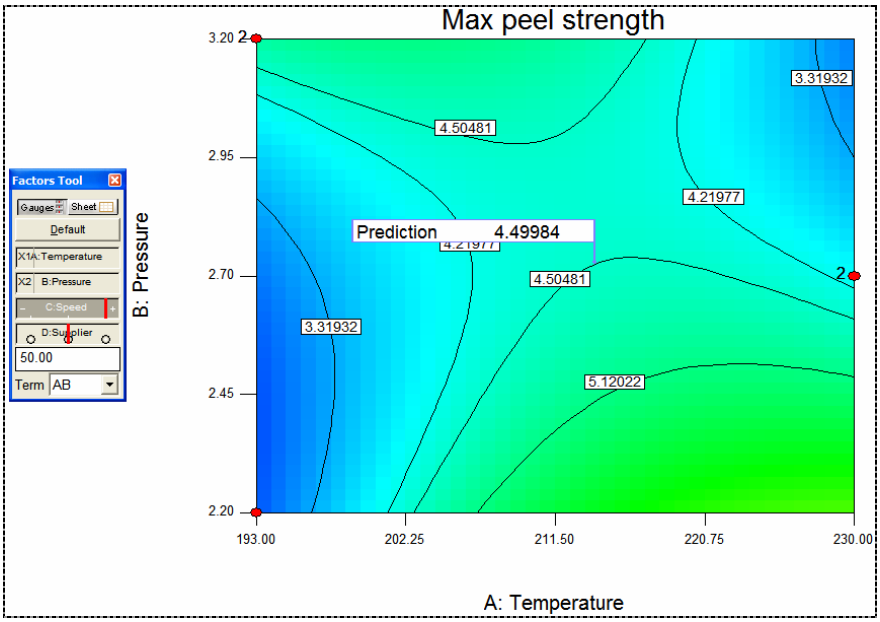
Supplier 1 not looking good, even at lowest speed (factor C)

On the **Factors Tool** for **D:Supplier** click the middle button see how S2 does. Then via a right-click on the graph **Add Contour** and drag it to where it comes close to the 4.5 required.



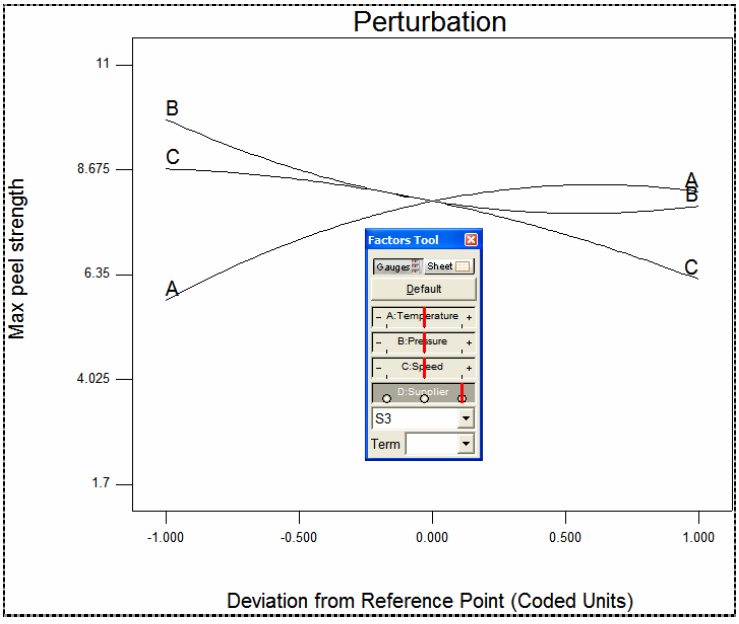
Contour plot for Supplier 2

On the **Factors Tool** click on **C:Speed** and slide it right. Then right-click and Add flag near the middle that achieves the response specification. This second supplier (S2) looks much better. With their material, the process need not be pushed too hard in one factor direction or another to meet the critical specification for maximum peel strength on the packaging material that's produced.



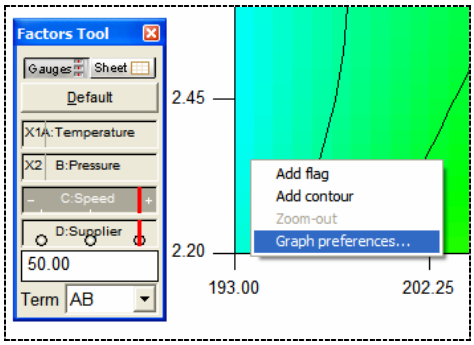
Supplier 2 looking good!

On the **Factors Tool** for **D:Supplier** click the right button see how S3 does – not good. ☹ Select **View**, **Perturbation** to get an overview on what's going on with this supplier. Click **Default** to center C:Speed. Then on the **Factors Tool** for **D:Supplier** click the right button to reset it for S3.



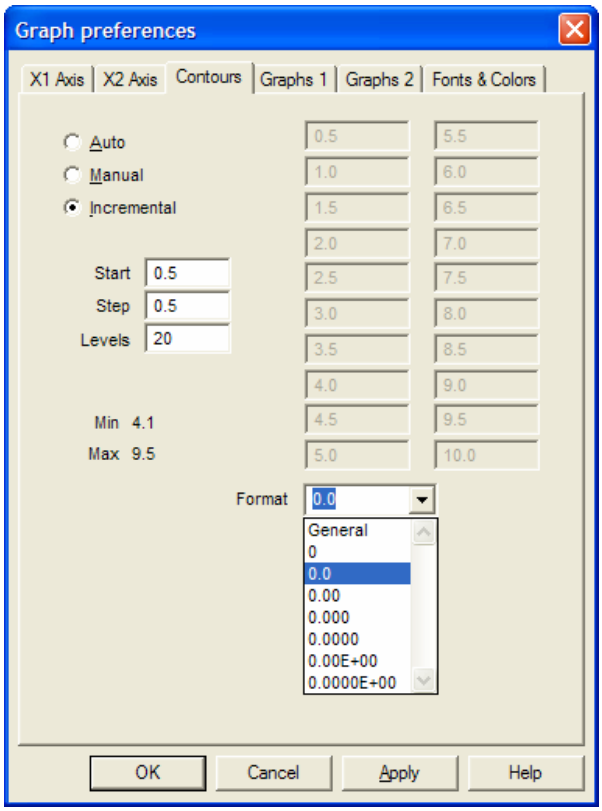
Perturbation plot for supplier S3

Notice that the high level of factor C might allow the 4.5 target to be achieved at some setting of A and B. Go back to **View**, **Contour** and on the **Factors Tool** click on **C:Speed** and slide it right. Then right-click over the graph and select **Graph preferences**.



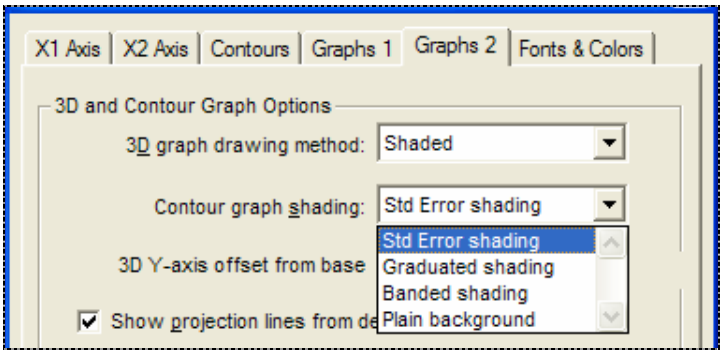
Third supplier chosen for contour plot with Graph preferences selected

On the **Contours** tab for graph preferences press the **Incremental** option and enter a **Start** of **0.5**, **Step** of **0.5** and set **Levels** at **20** (the maximum). Then change the **Format** to **0.0**.



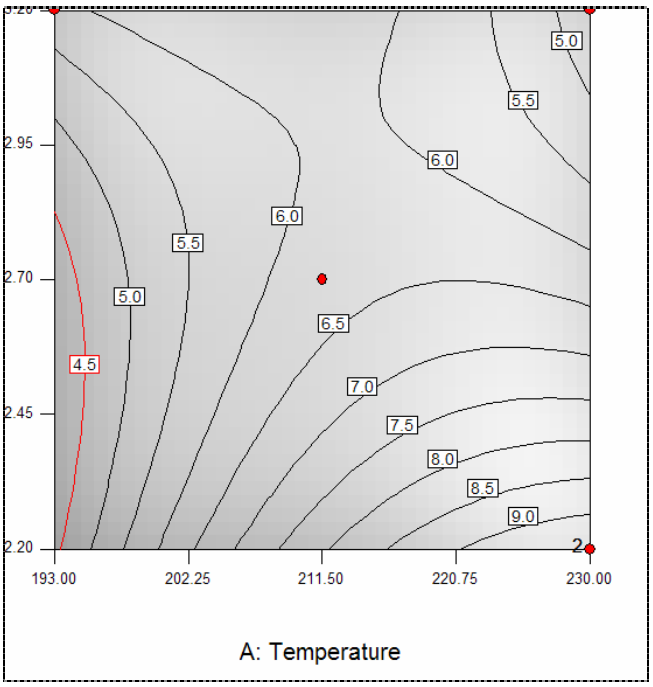
Changing the contour levels

On the Graphs 2 tab select for the Contour graph shading the Std error shading (better for black and white printing).



Standard error shading selected for contour graph

Press **OK** and note where the 4.5 specification is met by this supplier with the proper process setup.



The target of 4.5 highlighted as a contour for supplier 3

The red dots represent conditions where actual experimental runs were performed and, in one case, replicated (as indicated by the "2"). Unfortunately no points are seen near the desired contour. Thus this region is shaded more by Design-Expert to indicate a higher standard error for predictions.

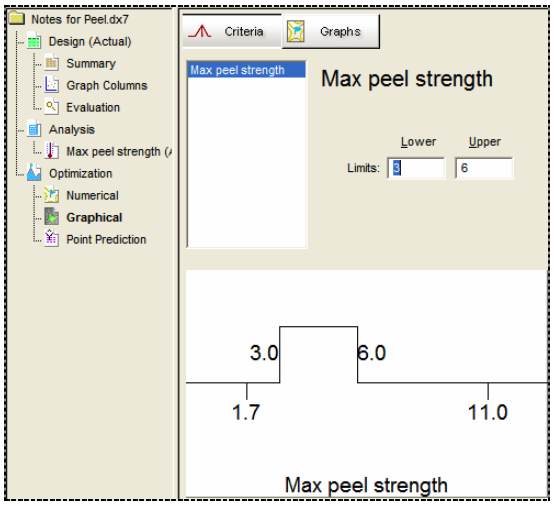
Graphical Optimization

Recall the questions asked at the outset of this tutorial about the new packaging machine developed by the engineers at P&G. We know the answer to the first one – yes, the process be adjusted to hit the maximum peel strength target of 4.5 lbf for one or more suppliers.

The purchase agent intended to break down the supply in this manner: 50% to S1, 25% to S2 and 25% to S3. This led to the second question: Can you find a process setup that will work for all suppliers? The contour plots for S1 made it clear that this supplier, despite being favored by the purchase agent, will not achieve the targeted peel strength within the current factor levels. That sinks the next most acceptable alternative: The two-supplier option with S1 being one of them. (For some reason, hopefully based on cost, the purchase agent really likes the first supplier!)

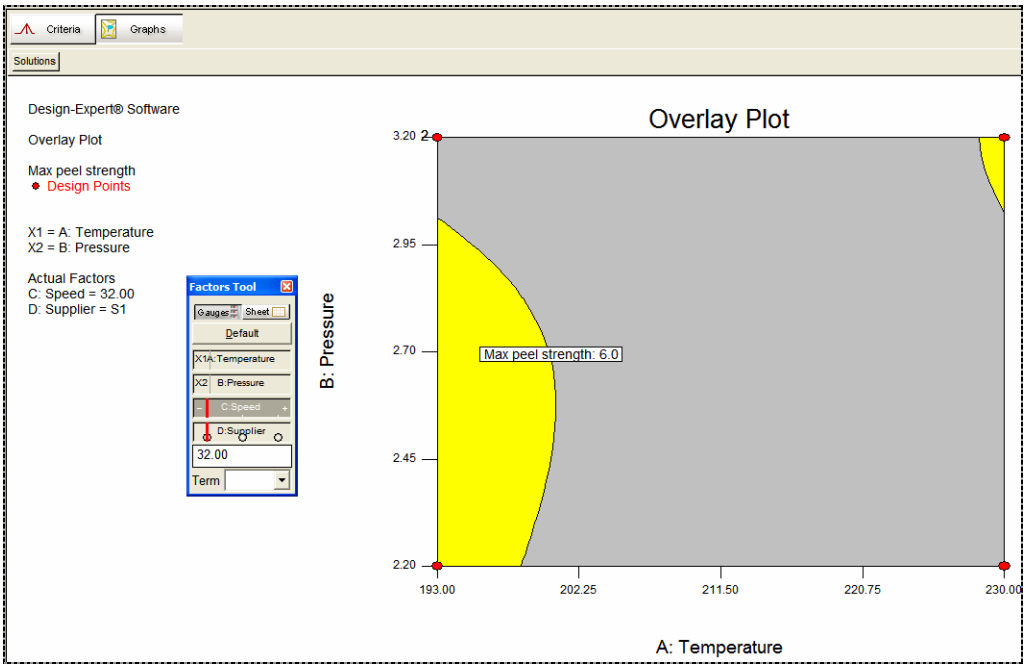
All that's left to the engineers is this fallback position: If it is not possible to achieve a common setup for even two of the three suppliers, perhaps the upper limit of 6 could be raised on the maximum peel strength specification. (Assume that customers are willing to use a scissors or, better yet, the package designers can add a notch in the plastic for easier opening.) Will this open up a window of operability for multiple suppliers at one set of process conditions?

It's time to explore the operating windows with each of the three suppliers for making packaging that must exceed 3 lbf (to prevent leaking) and not go above 6 lbf (making the package too difficult to open). Under the **Optimization** branch click the **Graphical** node. Enter for the **Limits** at **Lower 3** and **Upper 6**.



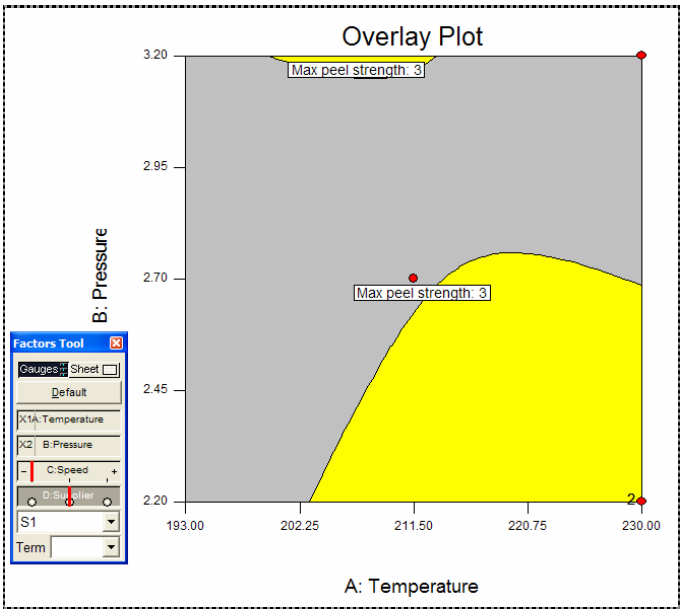
Criteria for graphical optimization

Click the ***Graphs*** button. There's no operating window at the default settings. On the **Factors Tool** you must click the bar for **C:Speed** and move it to the left, thus opening a sweet spot for the most-favored supplier one (S1).



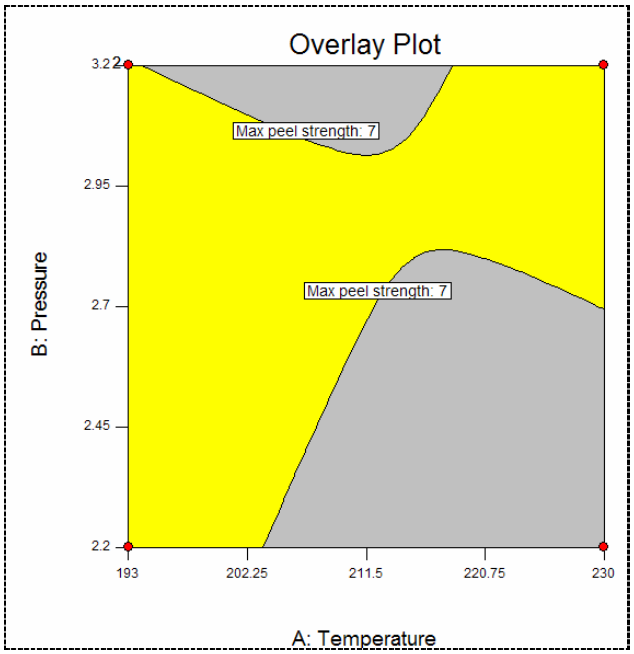
Sweet spot for supplier 1 at low speed (C)

Next click the **D:Supplier** buttons for the alternatives, S2 (middle) and S3 (right). Only S2 will work at this level of C, but it offers no window of opportunity for co-existing with S1.



Window for supplier 2 at same level of factor C

Now the only thing left to do is click the **D:Supplier** button for S1 (left) and click and drag the border for Max peel strength to the right, causing it to increase above 6. Go ahead and push it all the up to near 7. (Better have your scissors handy for opening the package!)



Dragging the border up to 7 for max peel strength

That's far enough into this case study for tutorial purposes. If you like, press through the other suppliers to see if there's a common condition where even the newly raised packaging specification can be achieved. The answer may surprise you.