Sizing Mixture (RSM) Designs for Adequate Precision via Fraction of Design Space (FDS)

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Sizing Mixture (RSM) Designs

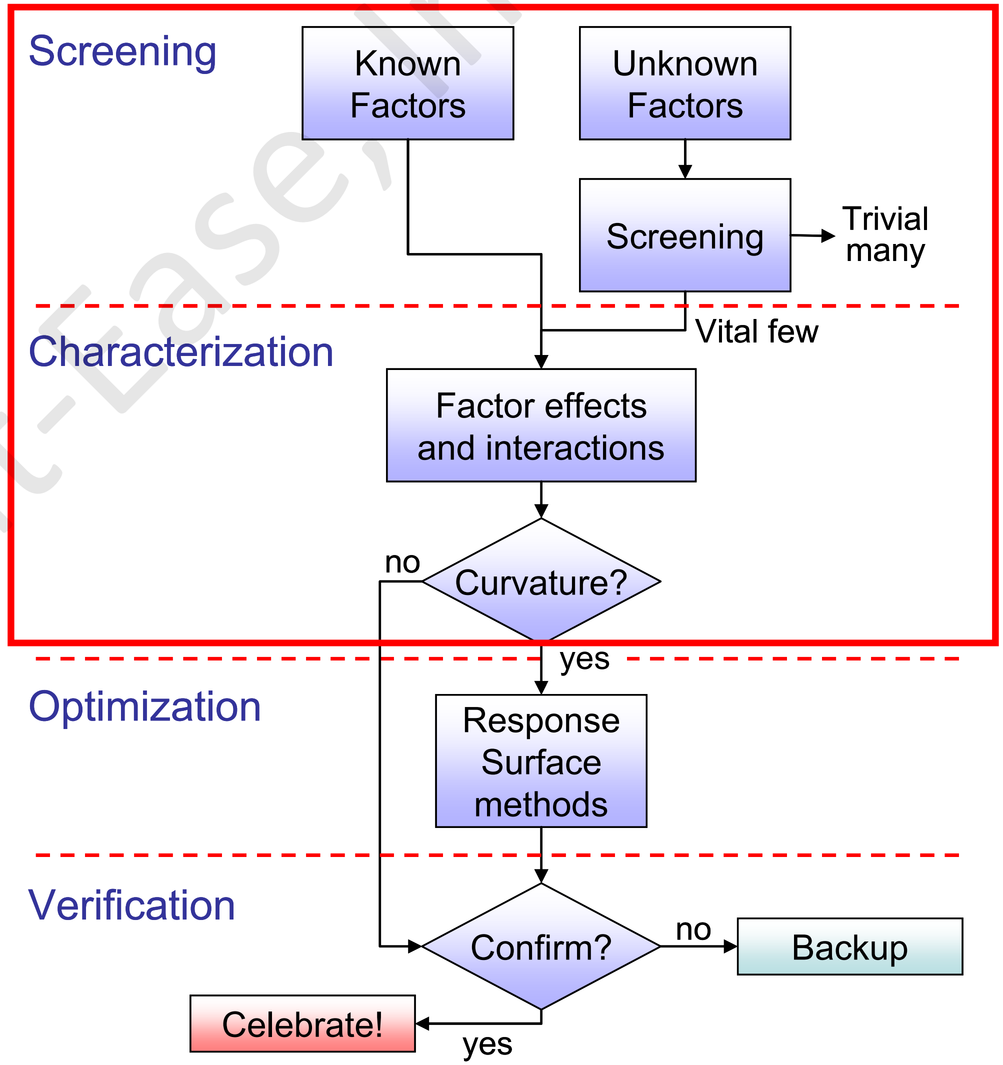
- **Review – Power to size factorial designs**
- Precision in place of power
  - Introduce FDS
- Sizing designs for precision
  - Two component mixture
  - Three component constrained mixture
- Sizing designs for prediction
- Sizing designs to detect a difference
- Summary

Sizing Factorial Designs

During screening and characterization the emphasis is on identifying factor effects.

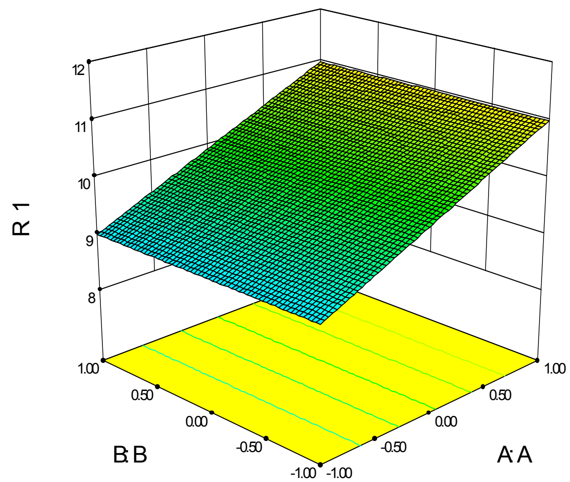
What are the important design factors?

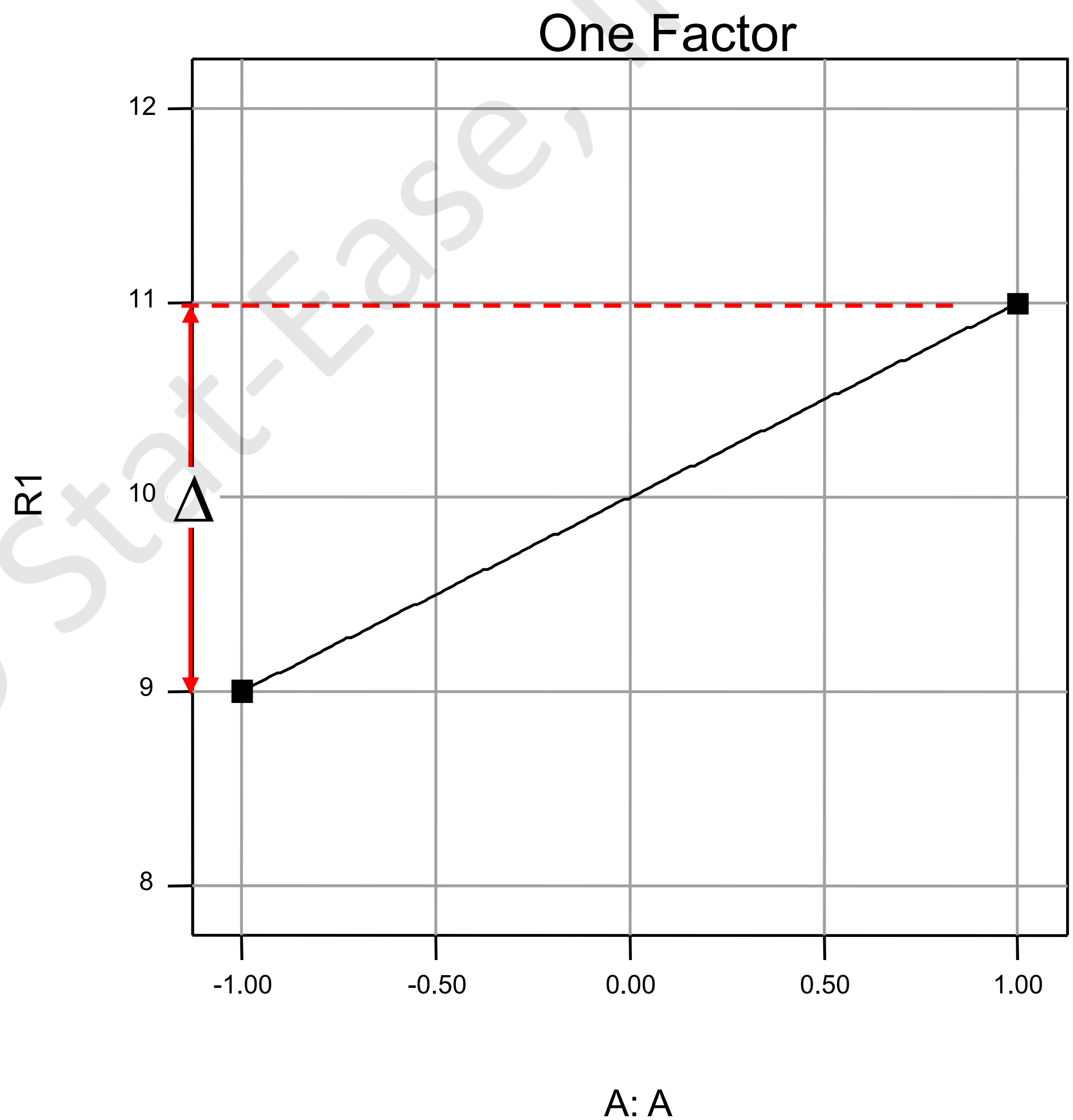
For this purpose power is an ideal metric to evaluate design suitability.



Factorial Design – Power

2^3 Full Factorial Δ=2 and σ=1





Factorial Design – Power

2^3 Full Factorial Δ=2 and σ=1

Leave Sigma and Delta fields blank to skip power calculation.

Responses: 1 (1 to 999) Edit model for power...

	Name	Units	Diff. to detect Delta("Signal")	Est. Std. Dev. Sigma("Noise")	Delta/Sigma (Signal/Noise Ratio)
	R1		2	1	2

Power is reported at a 5.0% alpha level to detect the specified signal/noise ratio. Recommended power is at least 80%.

R1

Signal (delta) = 2.00 Noise (sigma) = 1.00 Signal/Noise (delta/sigma) = 2.00

A	B	C
57.2 %	**57.2 %**	**57.2 %**

The following three slides dig into how power is computed.

One Replicate of 2^3 Full Factorial

$C = (X^TX)^{-1}$ matrix

The design determines the standard error of the coefficient:

$$C = \begin{pmatrix} 0.125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.125 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.125 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.125 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.125 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.125 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.125 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.125 \end{pmatrix}$$

$$\text{t-value}_i = \frac{\beta_i}{SE(\beta_i)} = \frac{\beta_i}{\sqrt{c_{ii}\hat{\sigma}^2}} = \frac{\beta_i}{\sqrt{(0.125)\hat{\sigma}^2}}$$

NonCentrality Parameter

2^3 Full Factorial Δ=2 and σ=1

The reference t distribution assumes the null hypothesis of $\Delta = 0$. The noncentrality parameter (2.828) defines the t distribution under the alternate hypothesis of $\Delta = 2$.

$$\text{noncentrality}_i = \frac{\beta_i}{\sqrt{c_{ii}\hat{\sigma}^2}} = \frac{\Delta_i / 2}{\sqrt{c_{ii}\hat{\sigma}^2}}$$

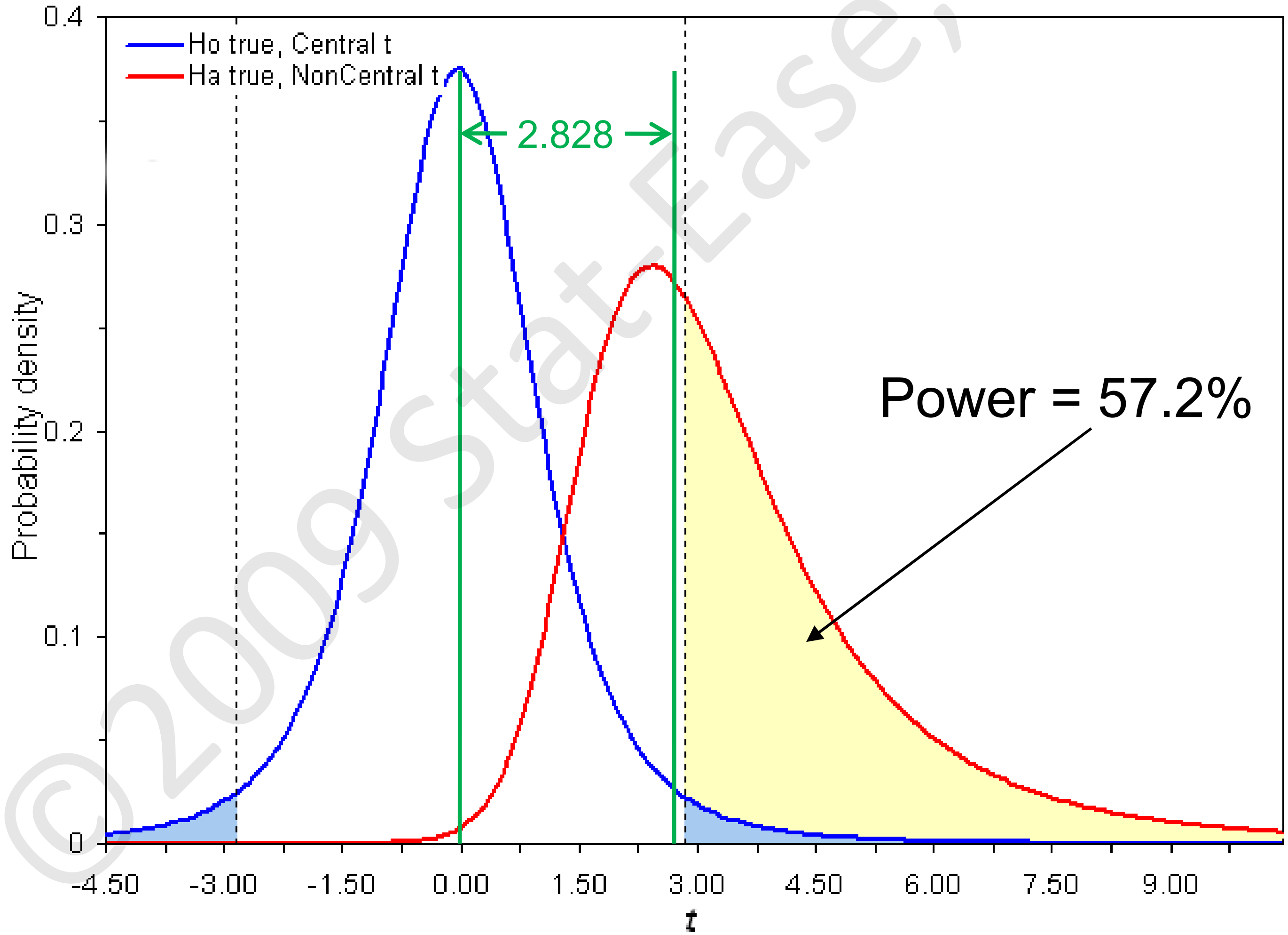
$$= \frac{1}{\sqrt{(0.125)(1)^2}}$$

$$= \frac{1}{0.3536} = 2.828$$

Factorial Design – Power

2^3 Full Factorial Δ=2 and σ=1

noncentral $t_{\alpha=0.05,df=4}$ with noncentrality parameter of 2.828



Factorial Design – Power

Two Replicates of 2^3 Full Factorial Δ=2 and σ=1

Leave Sigma and Delta fields blank to skip power calculation.

Responses: 1 (1 to 999) Edit model for power...

	Name	Units	Diff. to detect Delta("Signal")	Est. Std. Dev. Sigma("Noise")	Delta/Sigma (Signal/Noise Ratio)
	R1		2	1	2

Power is reported at a 5.0% alpha level to detect the specified signal/noise ratio. Recommended power is at least 80%.

R1

Signal (delta) = 2.00 Noise (sigma) = 1.00 Signal/Noise (delta/sigma) = 2.00

A	**B**	**C**
95.6 %	**95.6 %**	**95.6 %**

Two Replicates of 2^3 Full Factorial

$C = (X^TX)^{-1}$ matrix

The design determines the standard error of the coefficient:

$$C = \begin{pmatrix} 0.0625 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.0625 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0625 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.0625 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.0625 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.0625 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.0625 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0625 \end{pmatrix}$$

$$\text{t-value}_i = \frac{\beta_i}{SE(\beta_i)} = \frac{\beta_i}{\sqrt{c_{ii}\hat{\sigma}^2}} = \frac{\beta_i}{\sqrt{(0.0625)\hat{\sigma}^2}}$$

NonCentrality Parameter

Two Replicates of 2^3 Full Factorial Δ=2 and σ=1

$$\text{noncentrality}_i = \frac{\beta_i}{\sqrt{c_{ii}\hat{\sigma}^2}} = \frac{\Delta_i / 2}{\sqrt{c_{ii}\hat{\sigma}^2}}$$

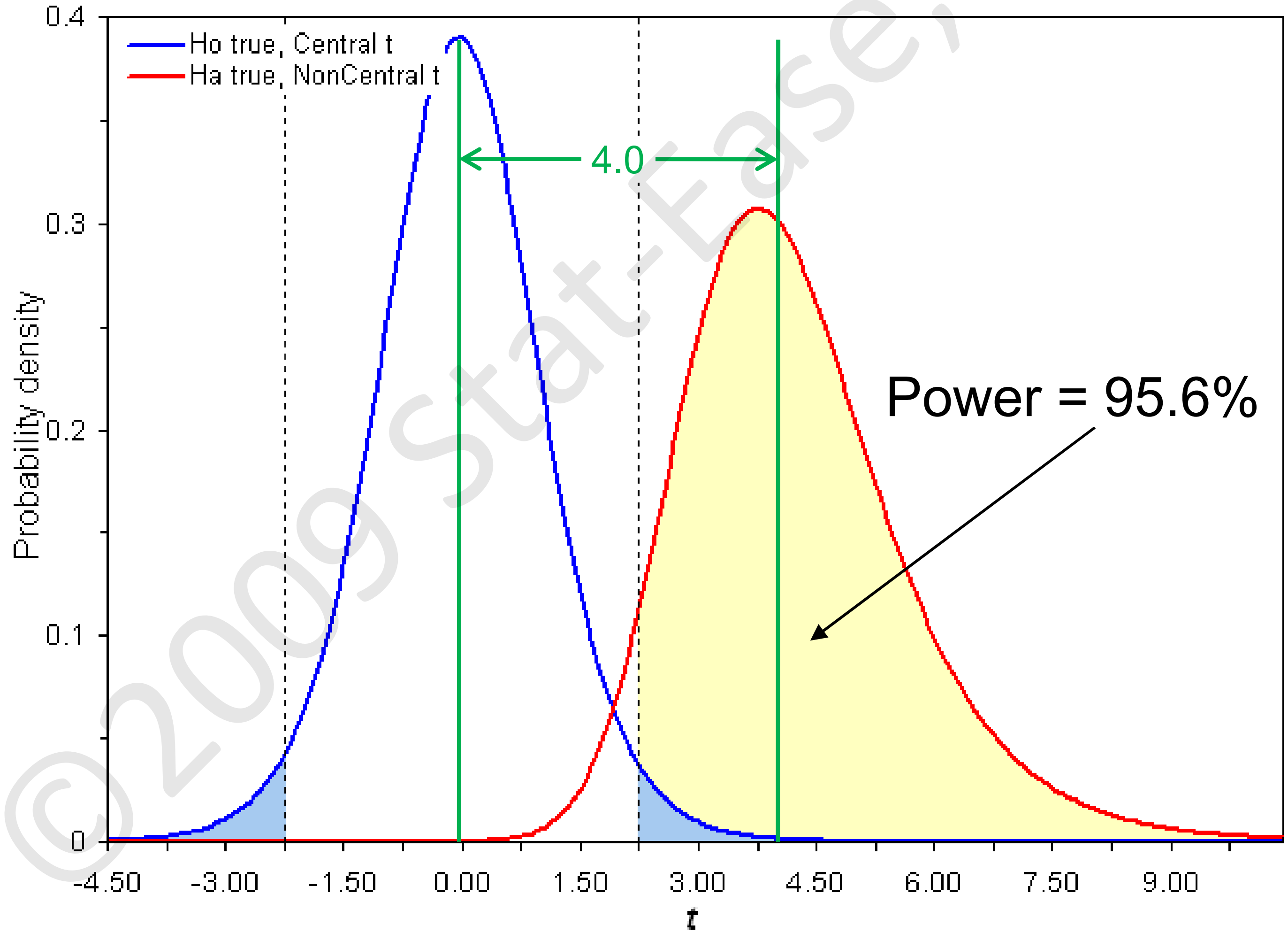
$$= \frac{1}{\sqrt{(0.0625)(1)^2}}$$

$$= \frac{1}{0.25} = 4.0$$

Factorial Design – Power

Two Replicates of 2^3 Full Factorial Δ=2 and σ=1

noncentral $t_{\alpha=0.05,df=12}$ with noncentrality parameter of 4.0



Conclusions

Factorial DOE
During screening and characterization (factorials) emphasis is on identifying factor effects. What are the important design factors? For this purpose power is an ideal metric to evaluate design suitability.

Sizing Mixture (RSM) Designs

- Review – Power to size factorial designs
- **Precision in place of power**
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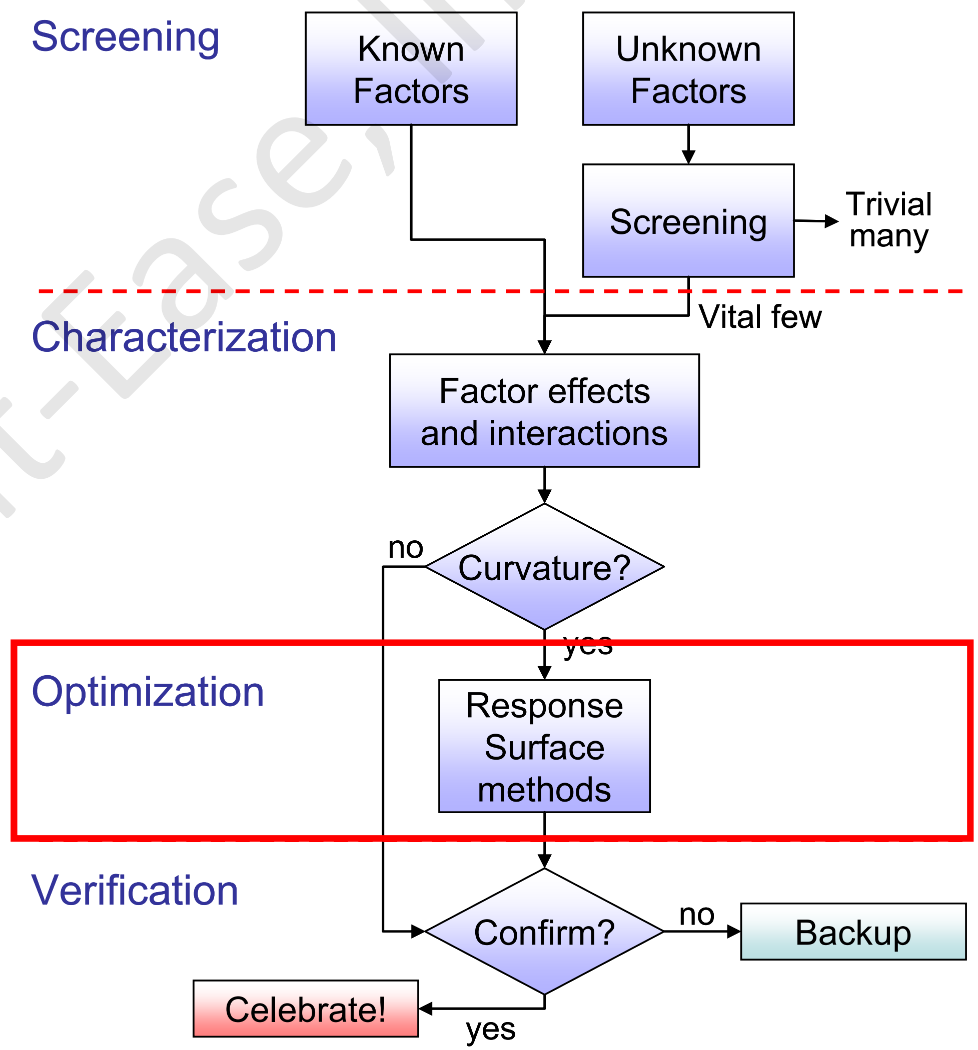
Sizing Response Surface Designs

When the goal is optimization emphasis is the fitted surface.

How well does the surface represent true behavior?

For this purpose precision (FDS) is an good metric to evaluate design suitability. *(Assuming model adequacy; i.e. insignificant lack of fit.)*



Design Properties

1. Estimate the designed for polynomial well.
2. Give sufficient information to allow a test for lack of fit.
   - ☑ Have more unique design points than coefficients in the model.
   - ☑ Provide an estimate of “pure” error.
3. Remain insensitive to outliers, influential values and bias from model misspecification.
4. Be robust to errors in control of the component levels.
5. Provide a check on model assumptions, e.g., normality of errors.
6. **Generate useful information throughout the region of interest, i.e., provide a good distribution of $\sqrt{\mathrm{Var}\left(\hat{Y}\right)/\sigma^2}$**
7. Do not contain an excessively large number of trials.

Fraction of **D**esign **S**pace:

- Calculates the volume of the design space having a prediction variance (PV) less than or equal to a specified value.
- The ratio of this volume to the total volume of the design volume is the fraction of design space.
- Produces a single plot showing the cumulative fraction of the design space on the x-axis (from zero to one) versus the PV on the y-axis.

Prediction Variance:

$$PV(x_0) = \frac{\operatorname{var}(\hat{y}_0)}{s^2} = x_0^T (X^T X)^{-1} x_0$$

PV is a function of:

- x_0 – the location in the design space (i.e. the x coordinates for all model terms).
- X – the experimental design (i.e. where the runs are in the design space).

Prediction standard error of the expected value:

$$PV\left(x_0\right) = \frac{\operatorname{var}\left(\hat{y}_0\right)}{s^2} = x_0^T\left(X^T X\right)^{-1} x_0$$

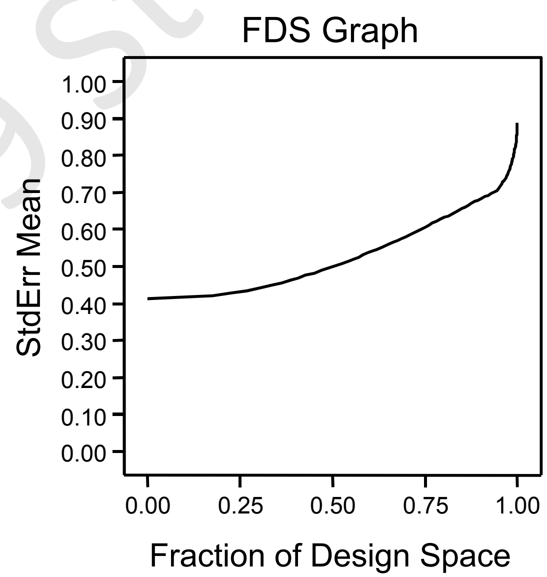
$$StdErr\left(x_0\right) = \frac{s_{\hat{y}_0}}{s} = \sqrt{PV\left(x_0\right)}$$

FDS Plot
Fraction of Design Space

1. Pick random points in the design space.
2. Calculate the standard error of the expected value

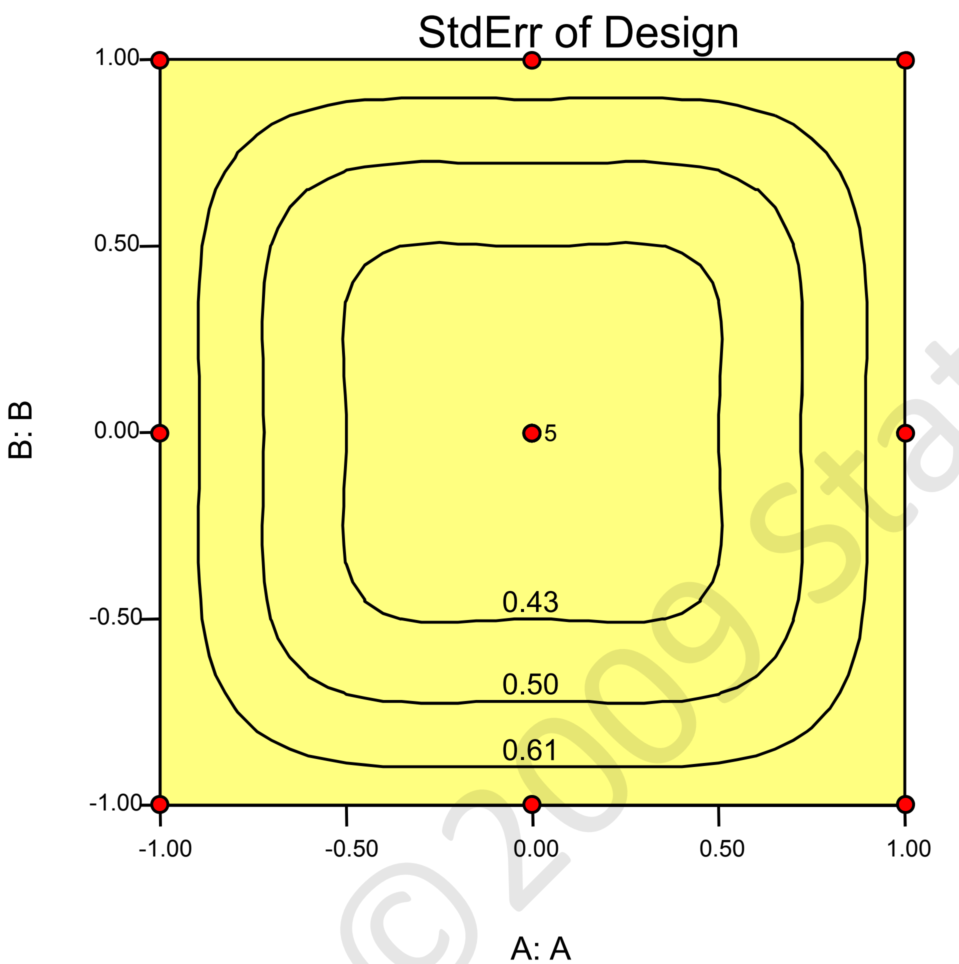
$$\frac{SE_{\hat{y}_0}}{s} = \sqrt{x_0^T \left(X^T X\right)^{-1} x_0}$$

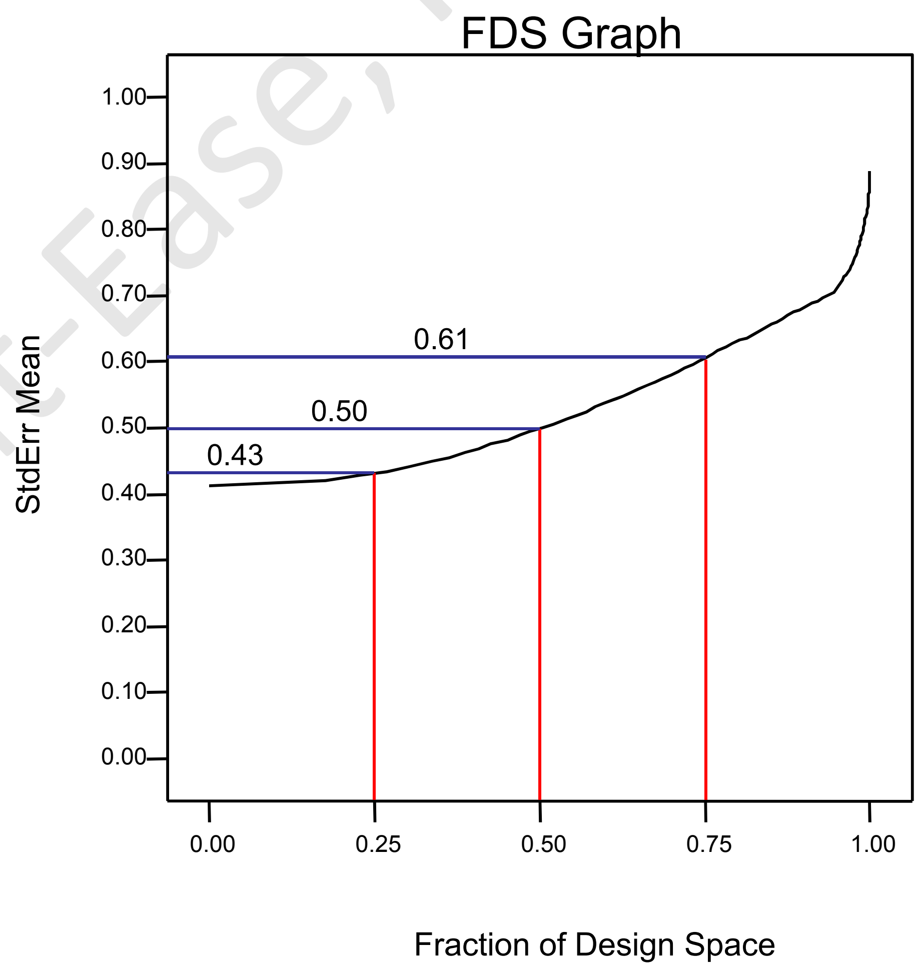
3. Plot the standard error as a fraction of the design space.



FDS – StdErr

Two-Factor Face Centered CCD





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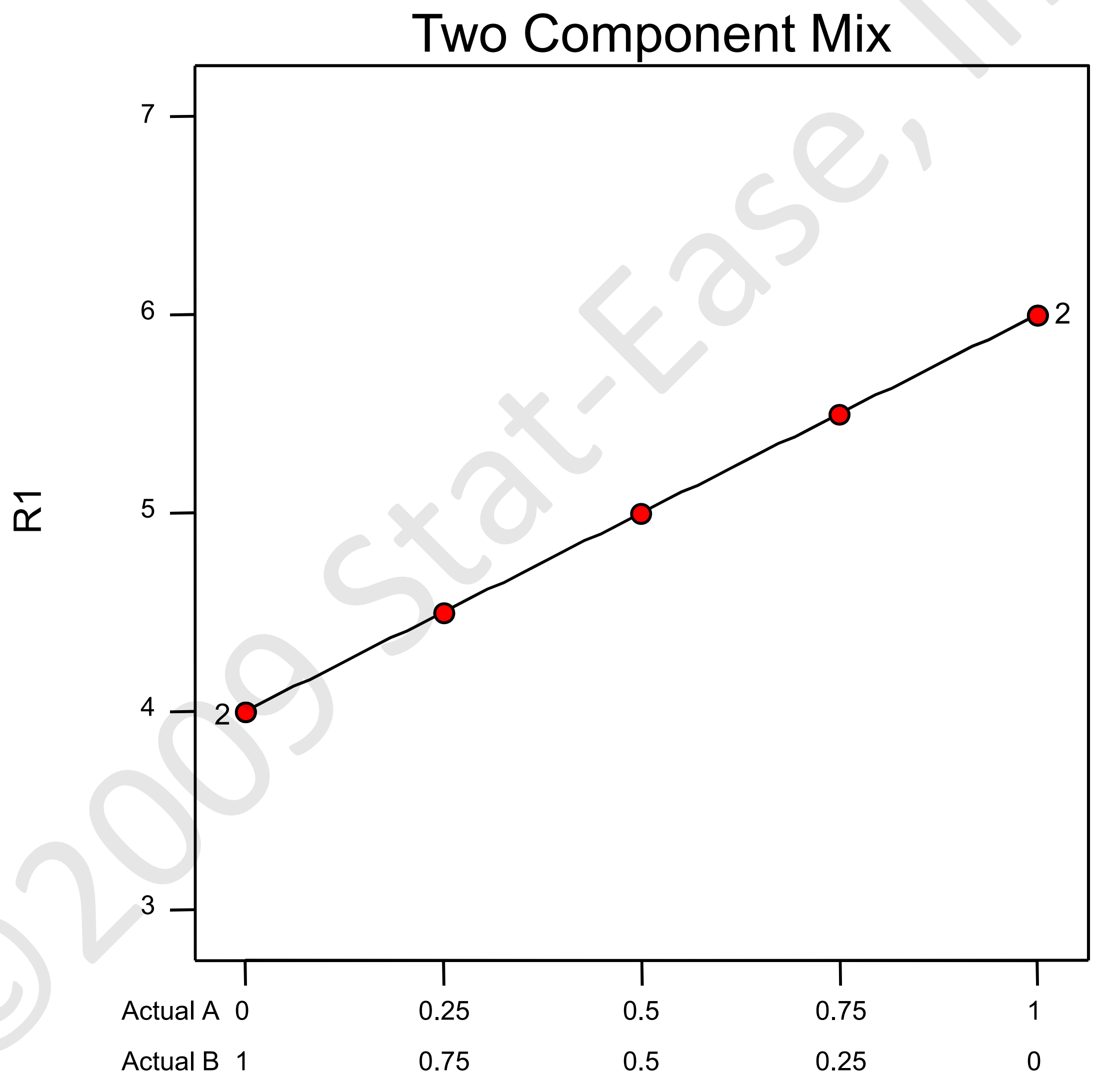
Extracted from MIX

Two Component Mixture

Linear Model



Extracted from MIX

Two Component Mixture

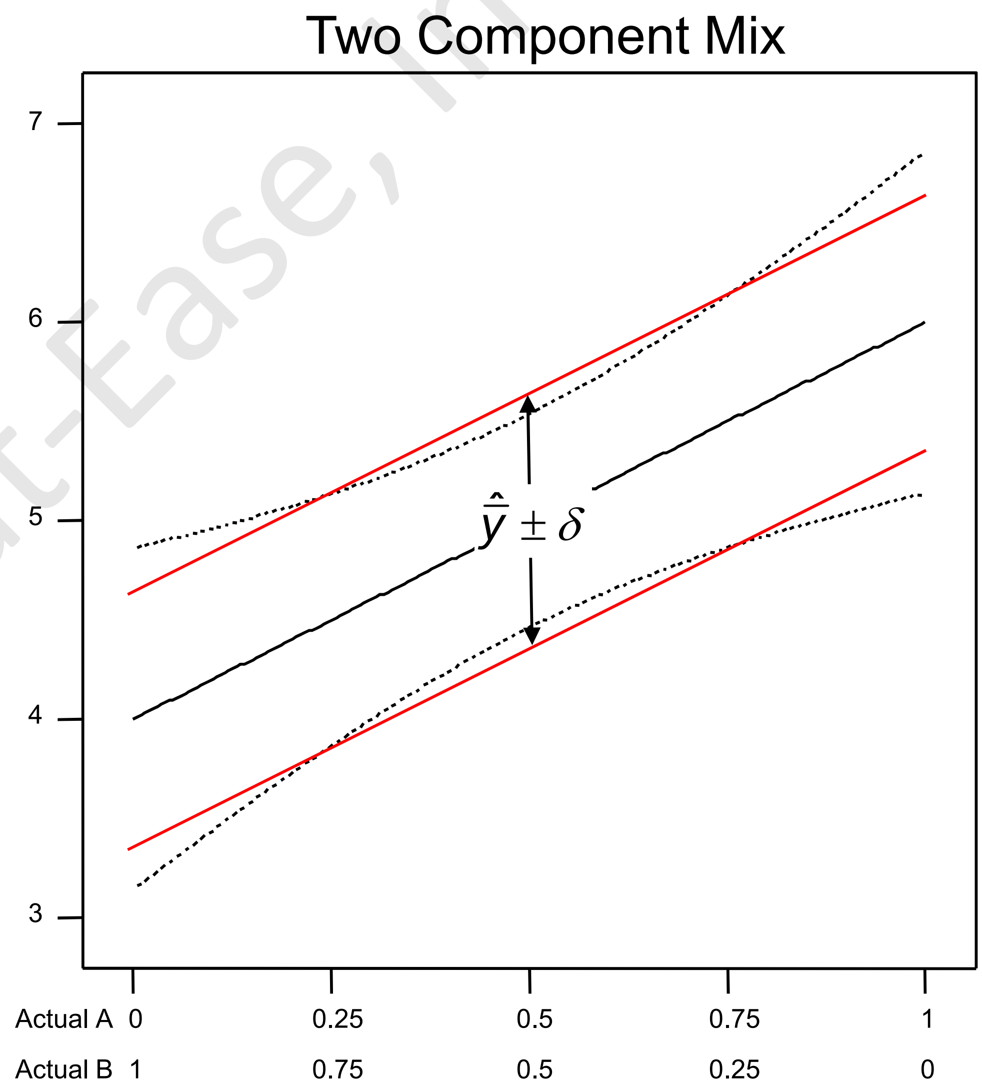
Linear Model

The solid center line is the fitted model; $\hat{y}$ is the expected value or mean prediction.

The curved dotted lines are the computer generated confidence limits, or the actual precision.

δ Is the half-width of the desired confidence interval, or the desired precision. It is used to create the outer straight lines.

Note: The actual precision of the fitted value depends on where we are predicting.

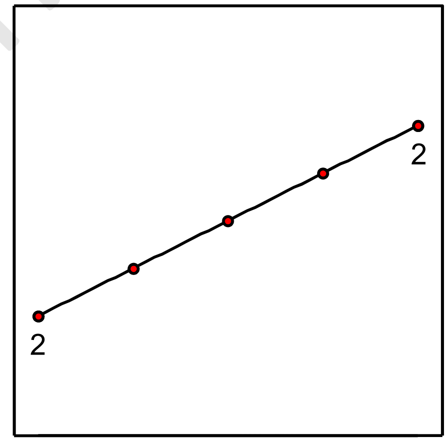


Extracted from MIX

Two Component Mixture
Linear Model

Build the two component mixture:

- Choose “**Simplex Lattice**”
- Continue >>
- Change the “**Order**” to “**Linear**” and the “**Replicates**” to “**2**”
- Continue >> Continue >>

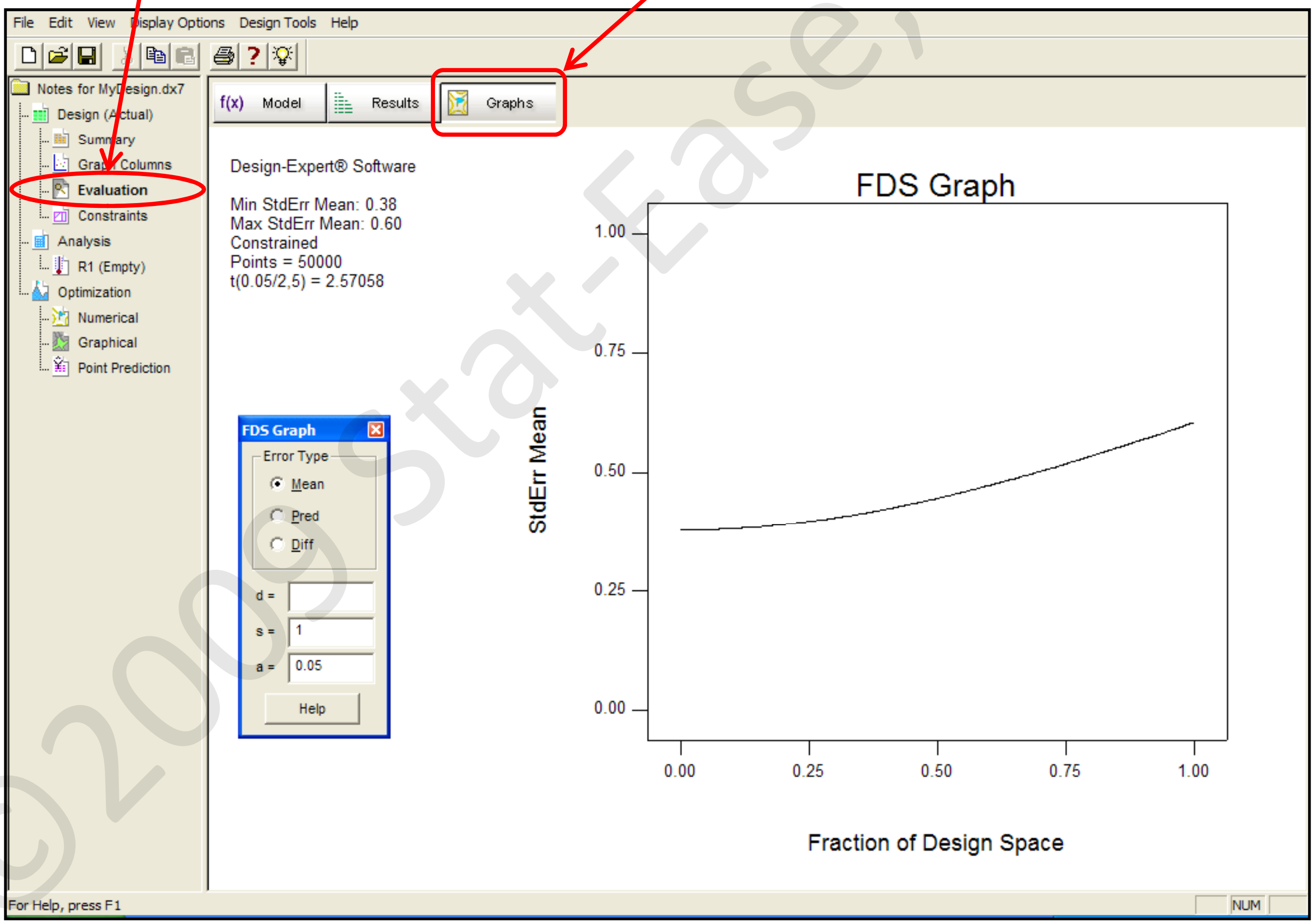


Extracted from MIX

Two Component Mixture

Linear Model

Click on "**Evaluate**" and then "**Graphs**":



Two Component Mixture

Linear Model

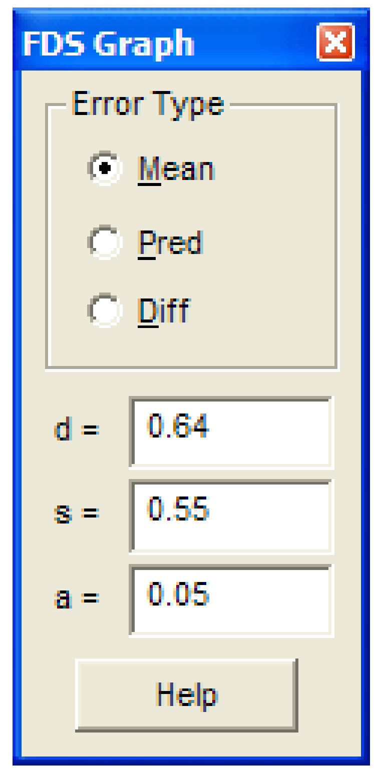
- Want a linear surface to represent the true response value within ± 0.64 with 95% confidence.
- The overall standard deviation for this response is 0.55.

Enter:

d (δ) = 0.64

s = 0.55

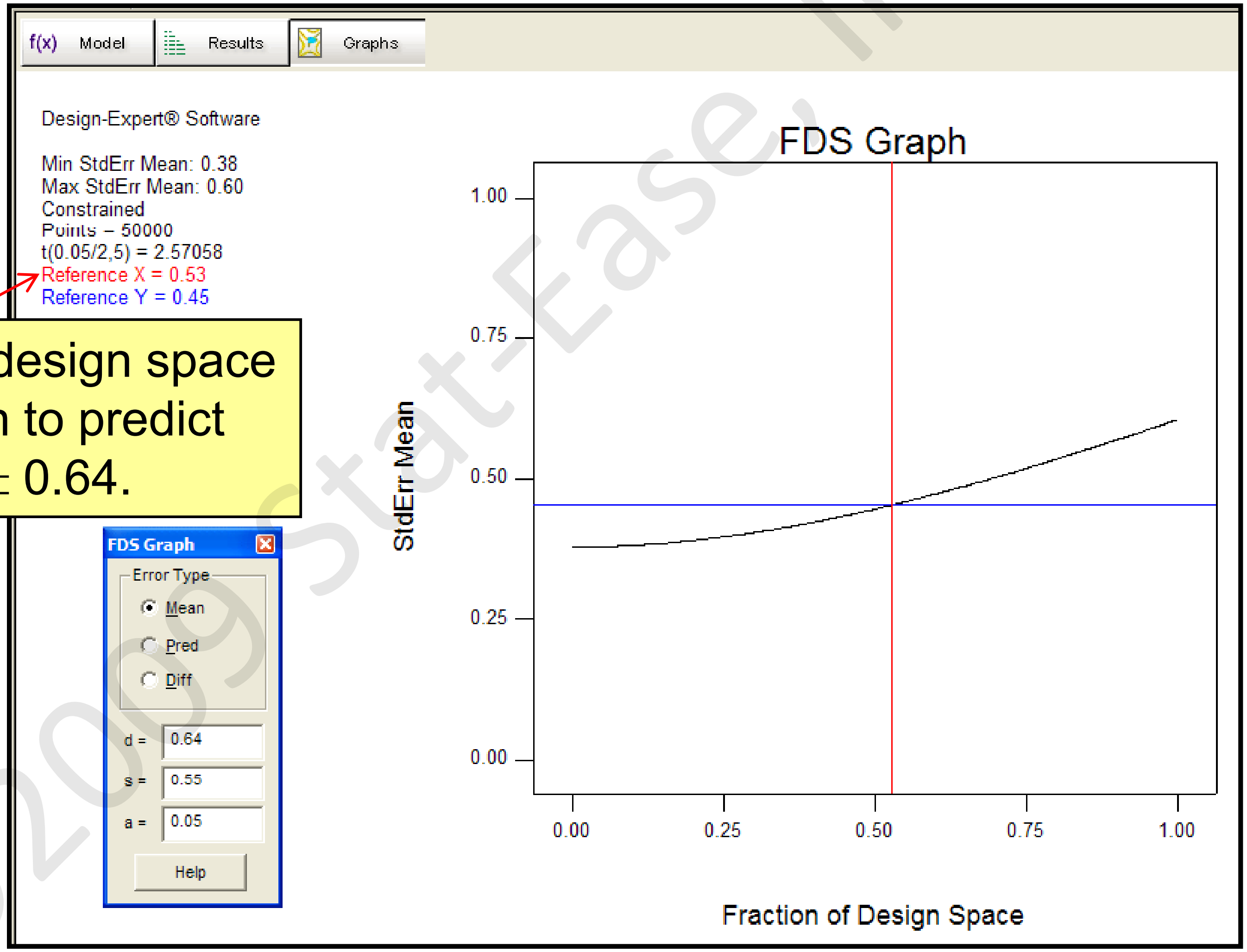
a (α) = 1 – 0.95 = 0.05



Extracted from MIX

Two Component Mixture

Linear Model



Only 53% of the design space is precise enough to predict the mean within ± 0.64.

Define Precision

Half Width of the Confidence Interval

Statistical Details

Confidence interval on the expected value:

The mean response is estimated and the precision of the estimate is quantified by a confidence interval:

$$\hat{\bar{y}} \pm t_{\alpha/2, df}\left(s_{\hat{\bar{y}}}\right)$$

We will use the half-width of the confidence interval (δ) to define the precision desired:

$$\delta = t_{\alpha/2, df}\left(s_{\hat{\bar{y}}}\right) \quad or \quad s_{\hat{\bar{y}}} = \frac{\delta}{t_{\alpha/2, df}}$$

What Precision is Needed?

Mean Confidence Interval Half-Width

Statistical Details

Half-width of confidence interval: δ

Input standard deviation estimate: s

$$\hat{\bar{y}} \pm \delta$$

$$\delta = t_{\alpha/2, df}\left(s_{\hat{\bar{y}}}\right)$$

$$s_{\hat{\bar{y}}} = s\sqrt{x_0^T \left(X^T X\right)^{-1} x_0}$$

$$StdErr\ Mean\left(FDS\right) = \frac{s_{\hat{\bar{y}}}}{s} = \sqrt{x_0^T \left(X^T X\right)^{-1} x_0}$$

Two Component Mixture

Linear Model

Statistical Details

- Want a linear surface to represent the true response value within ± 0.64 with 95% confidence.
- The overall standard deviation for this response is 0.55.

For 95% confidence $t_{.05/2,5} = 2.571,\ \delta = 0.64\ \ \&\ \ s = 0.55$

$$s_{\hat{y}} = \frac{\delta}{t_{\alpha/2,df}} = \frac{0.64}{2.571} = 0.25$$

$$StdErr\ Mean(FDS) = \frac{s_{\hat{y}}}{s} = \frac{0.25}{0.55} = 0.45$$

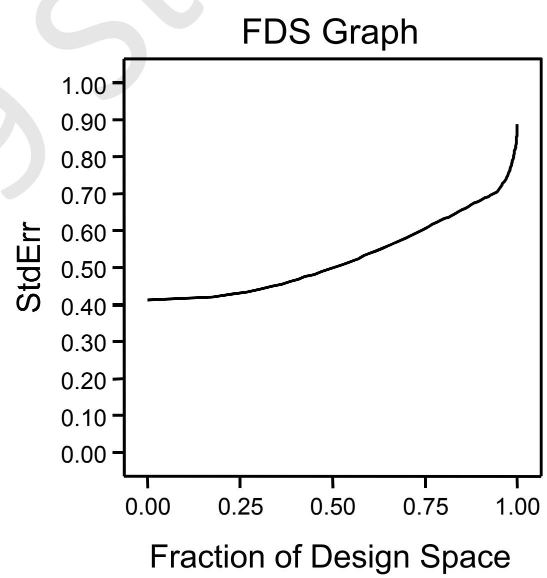
FDS – StdErr Mean Plot
Fraction of Design Space

Statistical Details

1. Pick random points in the design space.
2. Calculate the standard error of the expected value

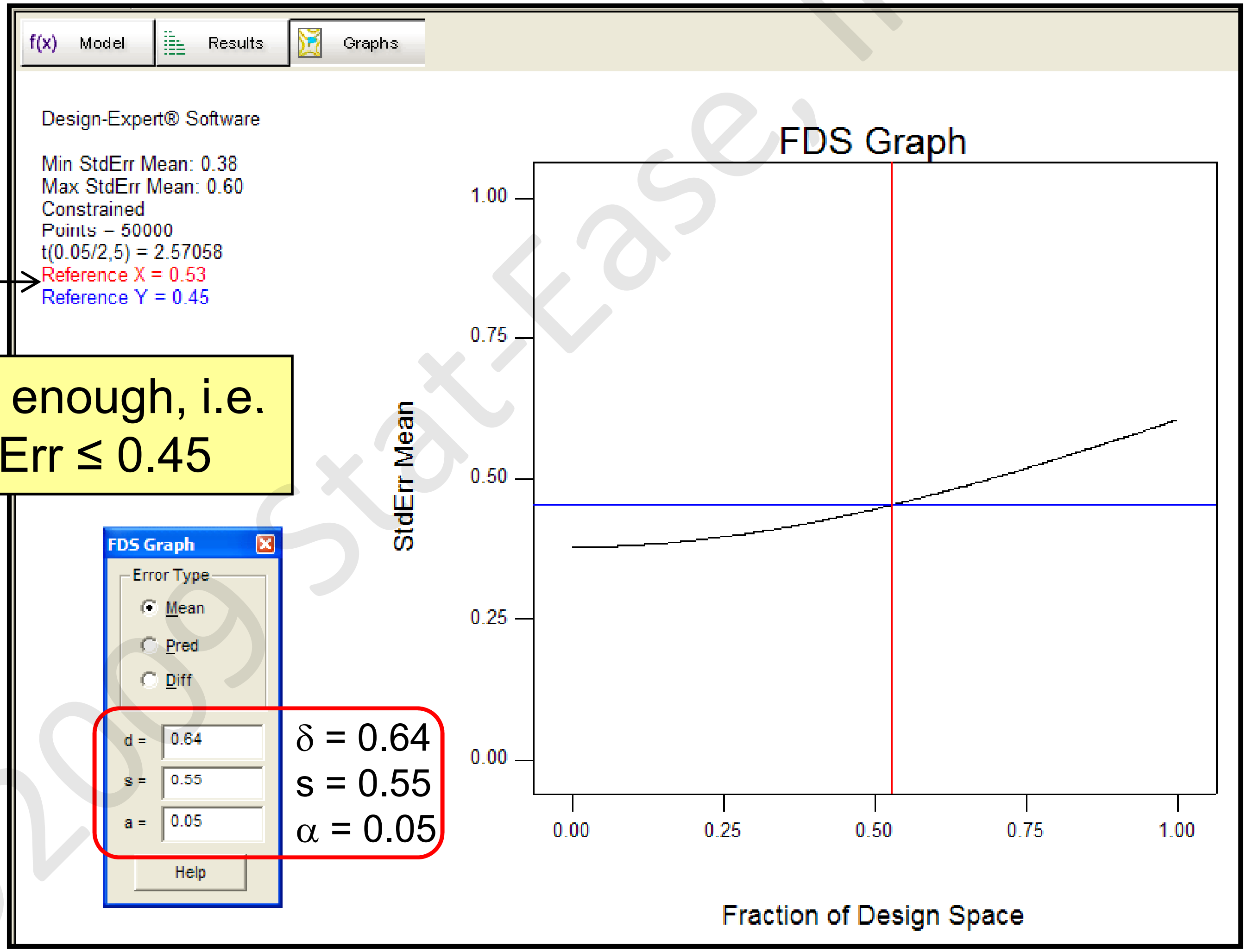
$$\frac{SE_{\hat{y}_0}}{s} = \sqrt{x_0^T \left(X^T X\right)^{-1} x_0}$$

3. Plot the standard error as a fraction of the design space.



Extracted from MIX

Two Component Mixture
Linear Model



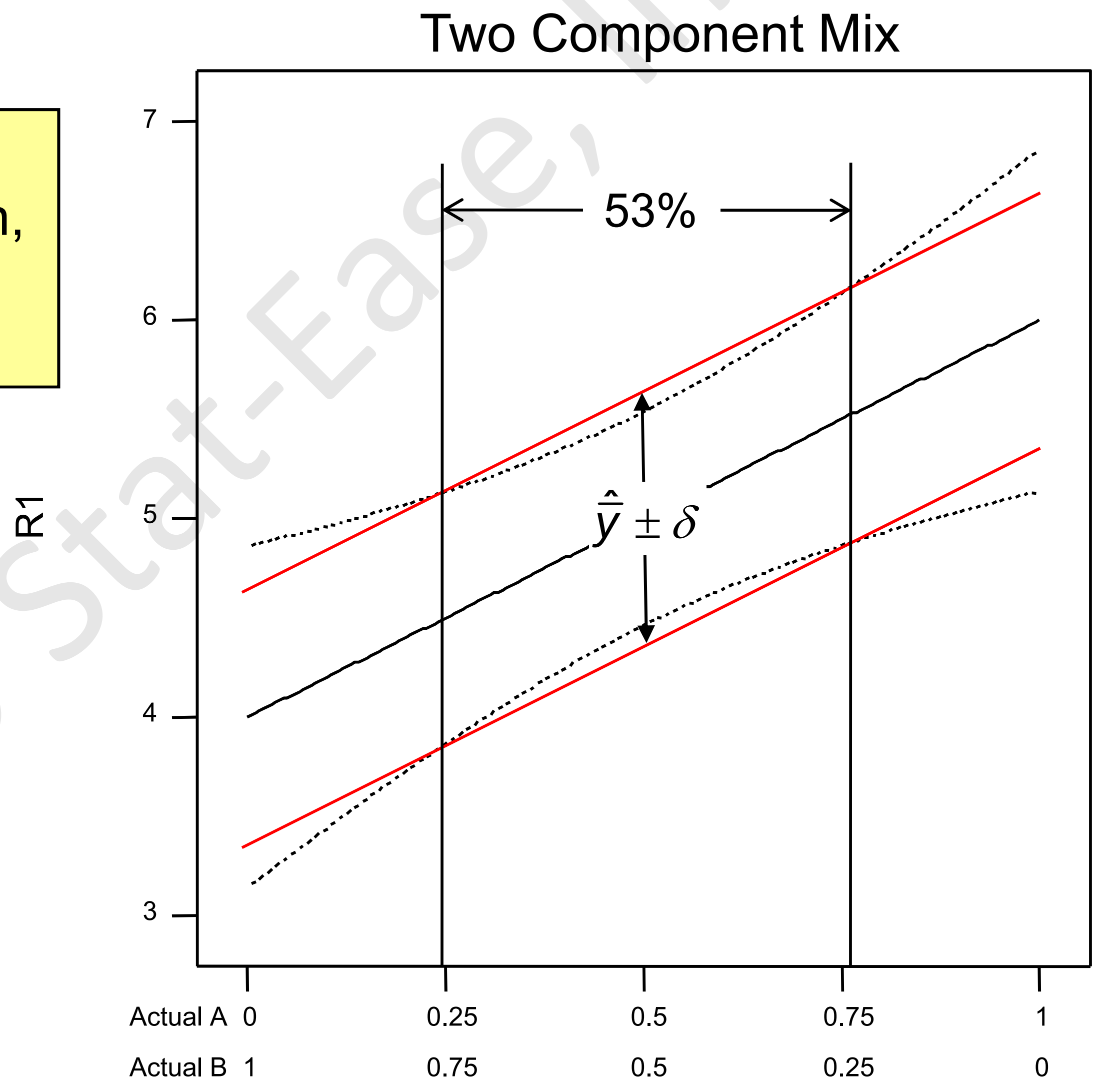
53% is precise enough, i.e.
53% has a StdErr ≤ 0.45

$\delta = 0.64$
$s = 0.55$
$\alpha = 0.05$

Two Component Mixture

Linear Model

53% of the design space has the desired precision, i.e. is inside the solid straight (red) lines.



Sizing for Precision

FDS ≥ ?

How good is good enough? Rules of thumb:

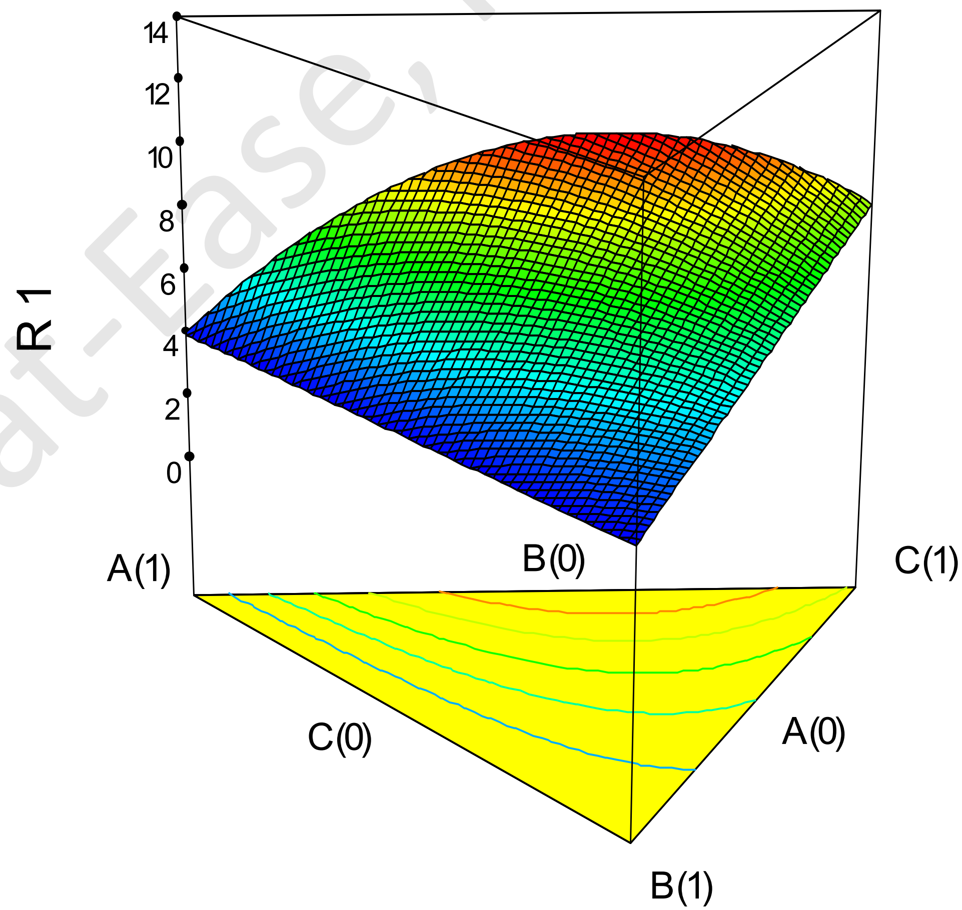
- For exploration want FDS ≥ 80%
- For verification want FDS ≥ 90% or higher!

What can be done to improve precision?

- Manage expectations; i.e. increase δ
- Decrease noise; i.e. decrease s
- Increase risk of Type I error; i.e. increase alpha
- Increase the number of runs in the design

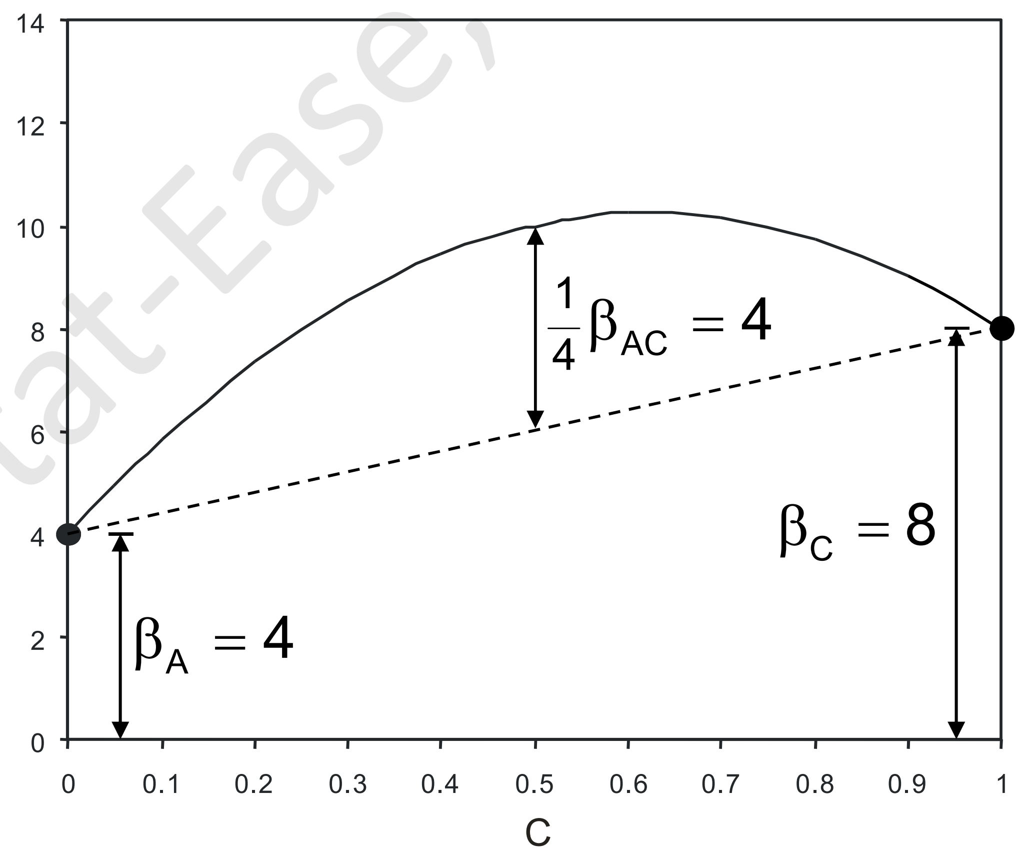
$$\hat{y} = 4A + 4B + 8C + 16AC$$

Most of the action occurs on the A-C edge because of the AC coefficient of 16. The quadratic coefficient of 16 means that the response is 4 units higher at A=0.5, C=0.5 than one would expect with linear blending.



$$\hat{y} = 4A + 4B + 8C + 16AC$$

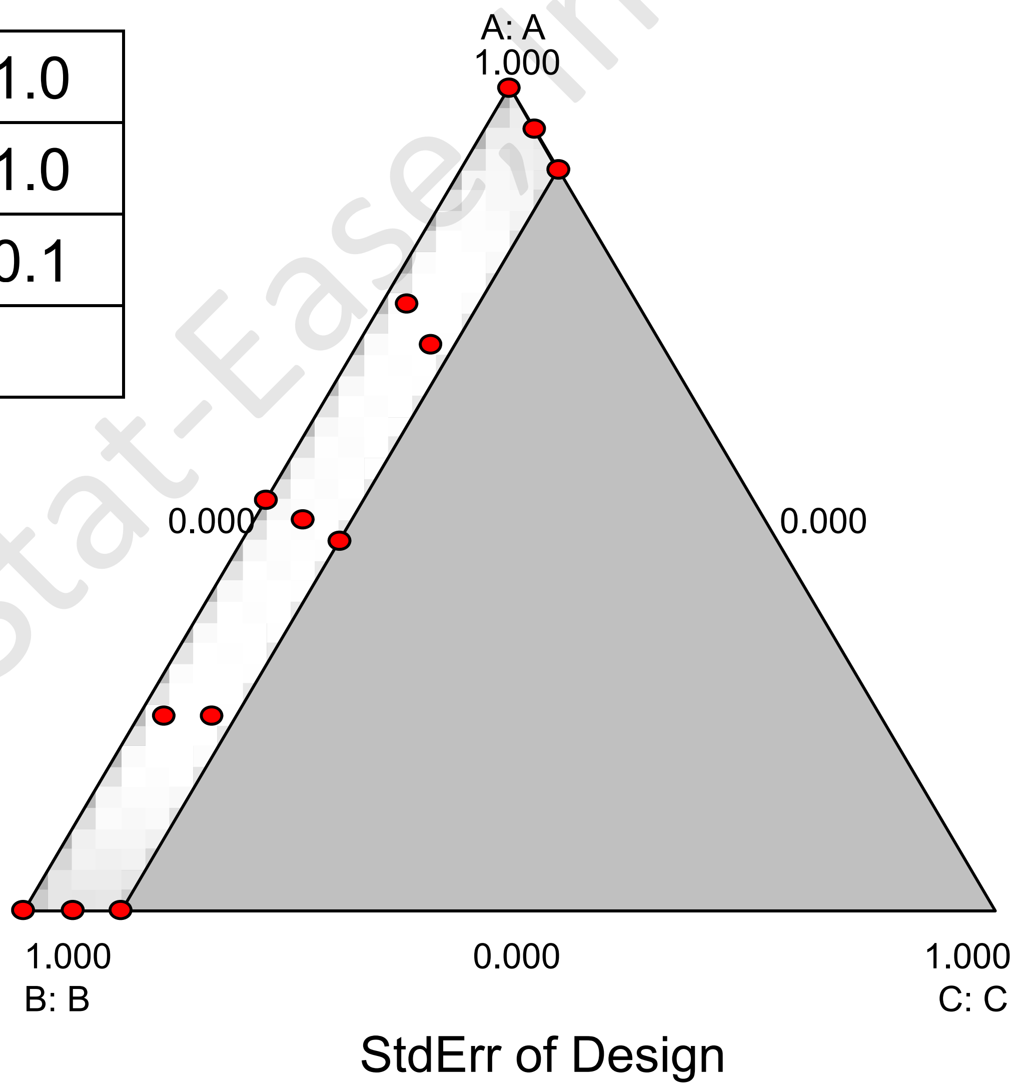
This illustrates the quadratic blending along the A-C edge as C increases from zero to one.



Mixture Constrained

Design for Quadratic Model

0.0	≤	A	≤	1.0
0.0	≤	B	≤	1.0
0.0	≤	C	≤	0.1
Total = 1.0				



Mixture Constrained

Quadratic Model (1 replicate)

Term	StdErr*	VIF	R_i-Squared	Power at 5 % alpha level for effect of 1 s	2 s	3 s
A	0.87	3.44	0.7094	5.0 %	5.0 %	5.0 %
B	0.87	3.44	0.7094	5.0 %	5.0 %	5.0 %
C	233.34	2722.29	0.9996	5.0 %	5.0 %	5.0 %
AB	2.86	2.19	0.5442	22.9 %	67.3 %	94.7 %
AC	258.98	1092.98	0.9991	5.0 %	5.0 %	5.0 %
BC	258.98	1092.98	0.9991	5.0 %	5.0 %	5.0 %

*Basis Std. Dev. = 1.0

Power is bottomed out at alpha!

Define Precision

Half Width of the Confidence Interval

6. Generate useful information throughout the region of interest.

Question: Will predictions, using the quadratic model from this design, be precise enough for our purposes?

- To know the truth requires an infinite number of runs; most likely this will exceed our budget.
- So the question is how precisely do we need to estimate the response?
- The trade off is more precision requires more runs.

Define Precision

Half Width of the Confidence Interval

Confidence interval on the expected value:

The mean response is estimated and the precision of the estimate is quantified by a confidence interval:

$$\hat{\bar{y}} \pm t_{\alpha/2,df}\left(s_{\hat{\bar{y}}}\right)$$

We will use half-width of the confidence interval (δ) to define the precision desired:

$$\delta = t_{\alpha/2,df}\left(s_{\hat{\bar{y}}}\right) \quad or \quad s_{\hat{\bar{y}}} = \frac{\delta}{t_{\alpha/2,df}}$$

What Precision is Needed?

Confidence Interval Half-Width

Half-width of confidence interval: δ

Input standard deviation estimate: s

$$s_{\hat{y}} = \frac{\delta}{t_{\alpha/2,df}}$$

$$s_{\hat{y}} = s\sqrt{x_0^T\left(X^T X\right)^{-1} x_0}$$

$$StdErr\ Mean\left(FDS\right) = \frac{s_{\hat{y}}}{s} = \sqrt{x_0^T\left(X^T X\right)^{-1} x_0}$$

Mixture Constrained

Quadratic Model (1 replicate)

- Want quadratic surface to represent the true response value within ±10 with 95% confidence.
- The standard deviation for this response is 7.8.

For 95% confidence $t_{.05/2,7} = 2.365,\ \delta = 10\ \ \&\ \ s = 7.8$

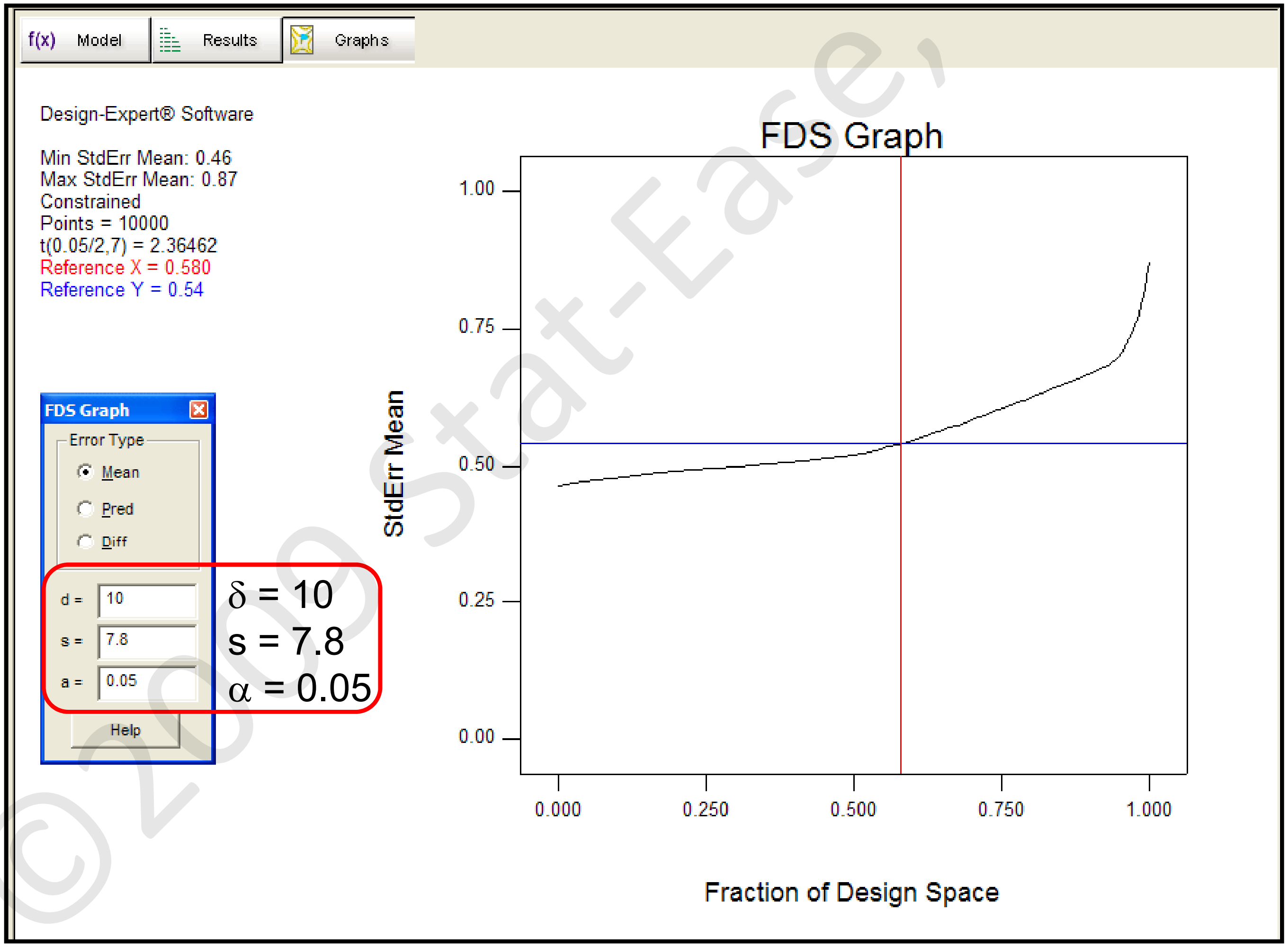
$$s_{\hat{y}} = \frac{\delta}{t_{\alpha/2,df}} = \frac{10}{2.365} = 4.23$$

$$StdErr\ Mean(FDS) = \frac{s_{\hat{y}}}{s} = \frac{4.23}{7.8} = 0.54$$

Mixture Constrained

Quadratic Model (1 replicate)

Only 58% of the design space has StdErr ≤ 0.54



Mixture Constrained

Quadratic Model (2 replicates)

				Power at 5 % alpha level for effect of		
Term	StdErr*	VIF	R_i-Squared	1 s	2 s	3 s
A	0.62	3.44	0.7094	5.0 %	5.0 %	5.0 %
B	0.62	3.44	0.7094	5.0 %	5.0 %	5.0 %
C	164.99	2722.29	0.9996	5.0 %	5.0 %	5.0 %
AB	2.02	2.19	0.5442	47.1 %	96.5 %	99.9 %
AC	183.12	1092.98	0.9991	5.0 %	5.0 %	5.0 %
BC	183.12	1092.98	0.9991	5.0 %	5.0 %	5.0 %

*Basis Std. Dev. = 1.0

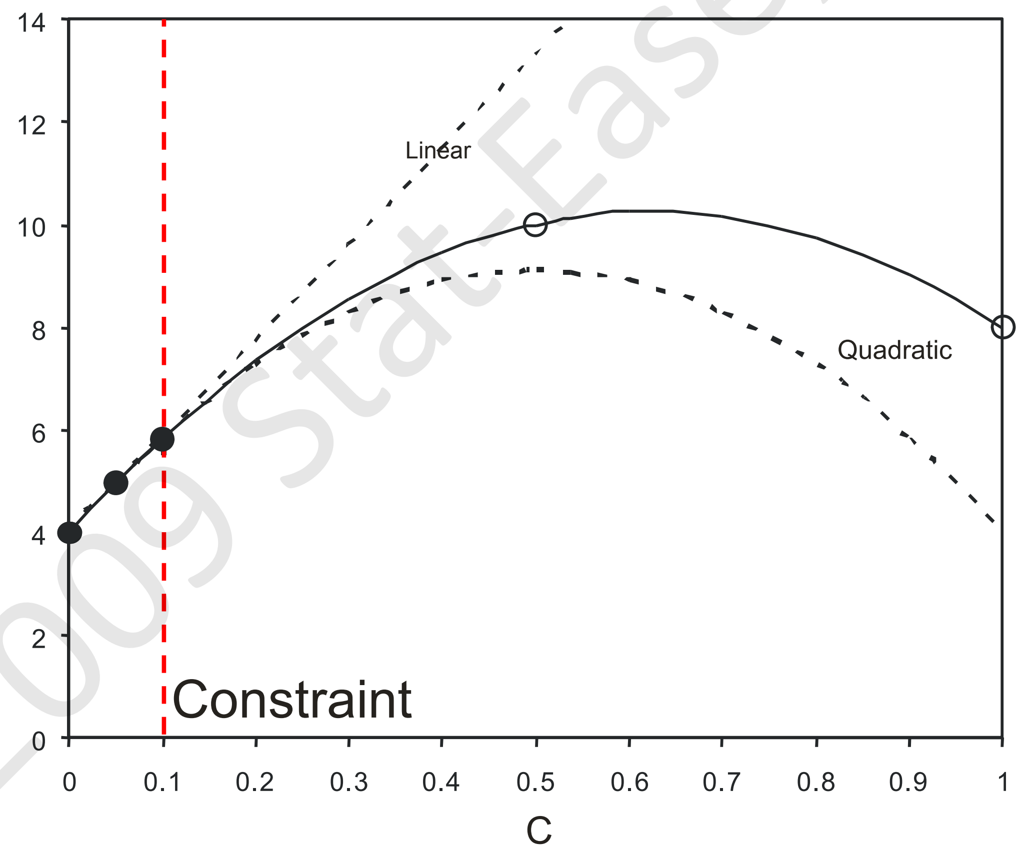
Adding a replicate does little to increase power!

Mixture Constrained

Why Does Power Not Increase?

Quadratic, linear or combinations of them model response.

Model Coefficients are Correlated!



Individual coefficients can not be resolved!

- Want quadratic surface to represent the true response value within ± 10 with 95% confidence.
- The overall standard deviation for this response is 7.8.

For 95% confidence $t_{.05/2,20} = 2.086$, $\delta = 10$ & $s = 7.8$

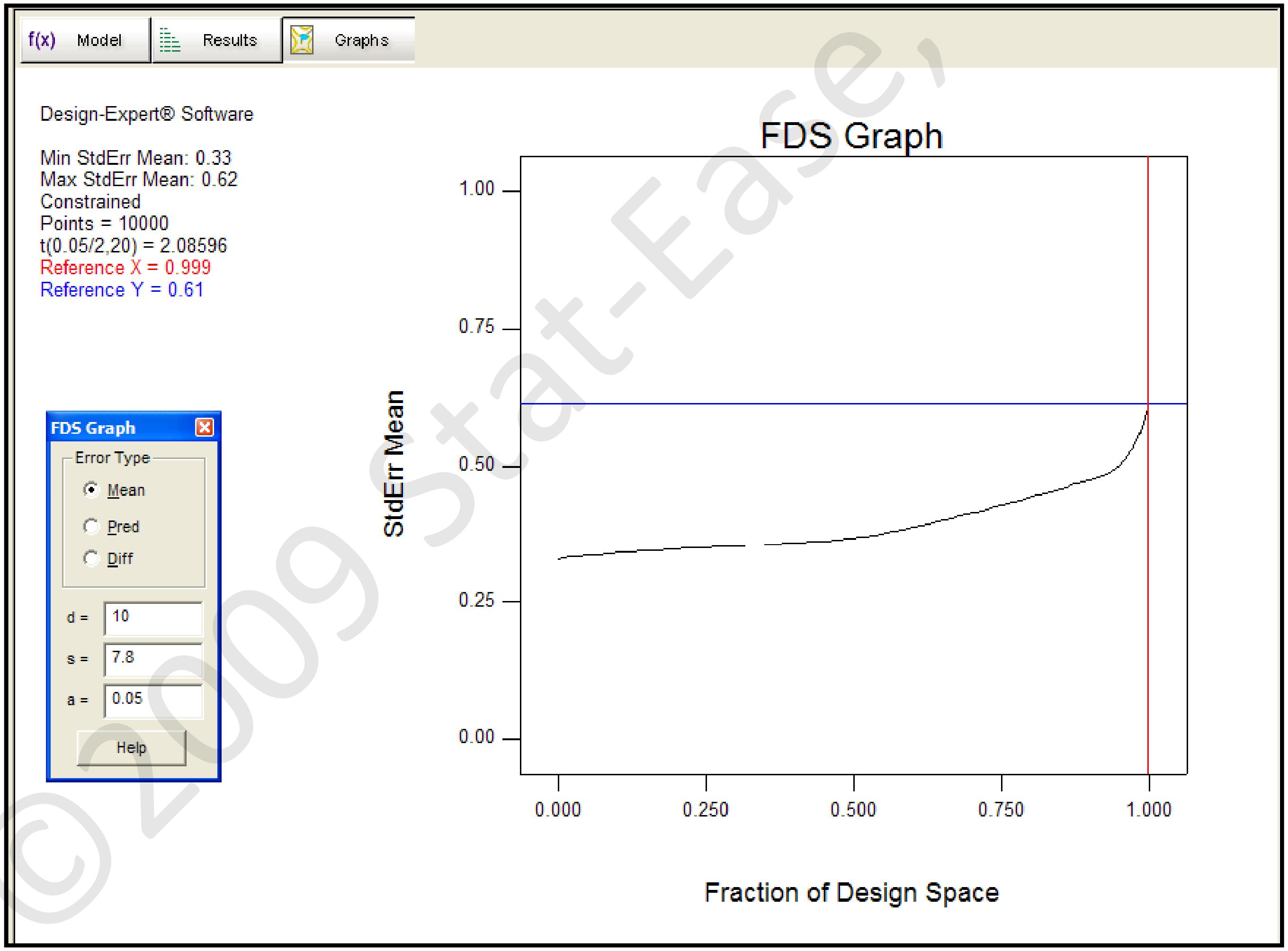
$$s_{\hat{\bar{y}}} = \frac{\delta}{t_{\alpha/2,df}} = \frac{10}{2.086} = 4.79$$

$$StdErr\ Mean(FDS) = \frac{s_{\hat{\bar{y}}}}{s} = \frac{4.79}{7.8} = 0.61$$

Mixture Constrained

Quadratic Model (2 replicates)

100% of the design space has StdErr ≤ 0.61



Precision Depends On

- The size of the confidence interval half-width (δ):

  A larger half-width (δ) increases the FDS.

- The size of the experimental error σ:

  A smaller σ increases the FDS.

- The α risk chosen:

  A larger α increases the FDS.

- Choose design appropriate to the problem:

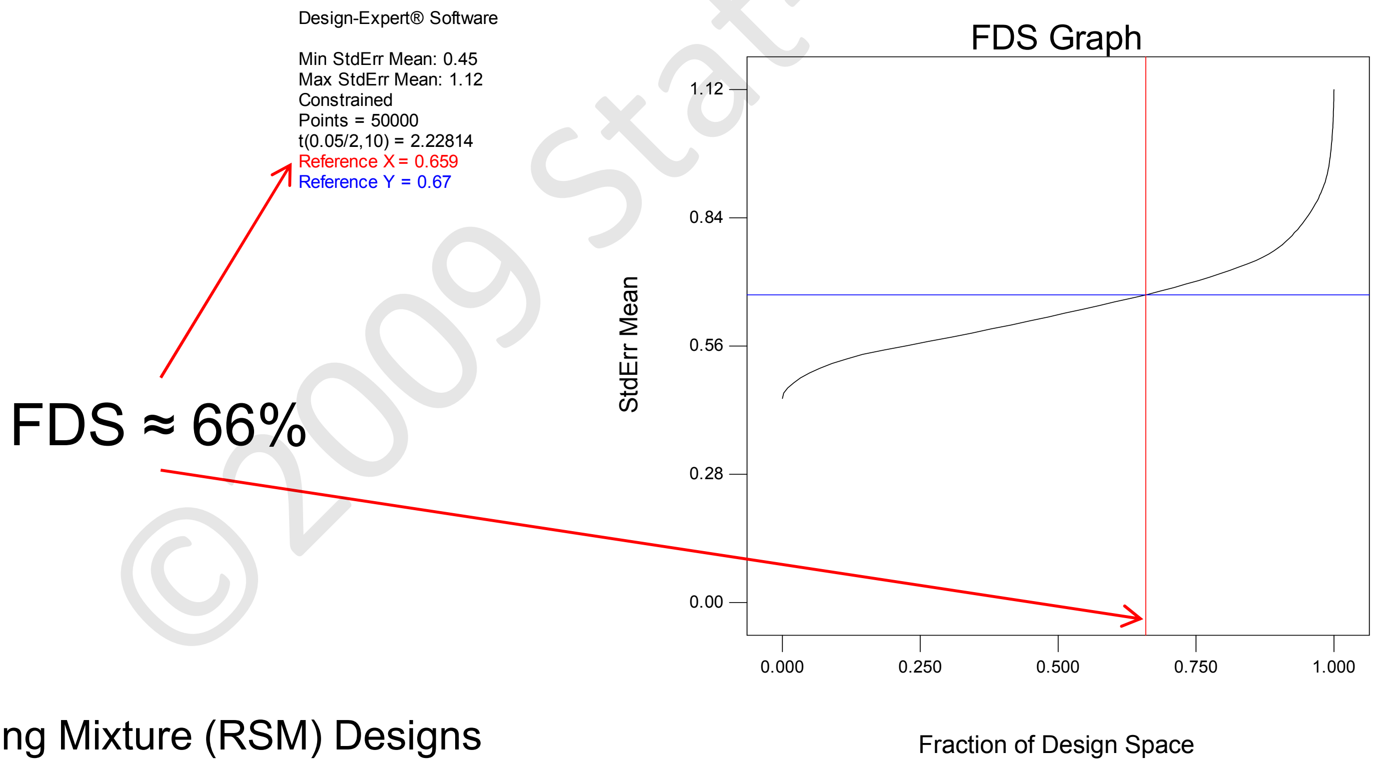
  Size the design for the precision required.

D-optimal Flare

Sizing for Precision

Does the design just built have adequate precision?

- Want FDS ≥ 80% with precision of ± 6
- Standard deviation for illumination (estimated from SPC data) is 4



D-optimal Flare
Sizing for Precision

To improve precision what type of points should be added; model, lack-of-fit or replicates?

Answer: Increasing model points is the most effective way to improve precision. Assuming there are already 5 each for lack of fit and replicates, then increase the number of model points.

Extracted from MIX

D-optimal Flare

Sizing for Precision

Rebuild your design increasing the number of model points one at a time until you achieve a FDS ≥ 80% for d = 6 and s = 4.

- When rebuilding your design say:
  - "**No**" to "Save changes…?"
  - "**Yes**" to "Use previous design info?"
- The default was 14 model points; so try 15, then 16, then 17, etc.
- How many "extra" model points did you add?

Answer: 17 – 14 = 3 extra pts

Conclusions

Factorial DOE	Mixture Design and Response Surface Methods
During screening and characterization (factorials) emphasis is on identifying factor effects.	When the goal is optimization (usually the case for mixture design & RSM) emphasis is on the fitted surface.
What are the important design factors?	How well does the surface represent true behavior?
For this purpose power is an ideal metric to evaluate design suitability.	For this purpose precision (FDS) is a good metric to evaluate design suitability.

Sizing Mixture (RSM) Designs

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Prediction standard error of a new observation:

$$PV\left(x_0\right)=\frac{\text{var}\left(\hat{y}_0\right)}{s^2}=\left(1+x_0^T\left(X^TX\right)^{-1}x_0\right)$$

$$StdErr\ Pred\left(x_0\right)=\frac{s_{\hat{y}_0}}{s}=\sqrt{PV\left(x_0\right)}$$

Define Precision

Half Width of the Prediction Interval

Prediction interval on a future observation:

The response is estimated and the precision of the estimate is quantified by a prediction interval:

$$\hat{y} \pm t_{\alpha/2, df}\left(s_{\hat{y}}\right)$$

We will use half-width of the prediction interval (δ) to define the precision desired:

$$\delta = t_{\alpha/2, df}\left(s_{\hat{y}}\right) \quad or \quad s_{\hat{y}} = \frac{\delta}{t_{\alpha/2, df}}$$

What Precision is Needed?

Prediction Interval Half-Width

Half-width of prediction interval: δ

Input standard deviation estimate: s

$$s_{\hat{y}} = \frac{\delta}{t_{\alpha/2, df}}$$

$$s_{\hat{y}} = s\sqrt{1 + x_0^T \left(X^T X\right)^{-1} x_0}$$

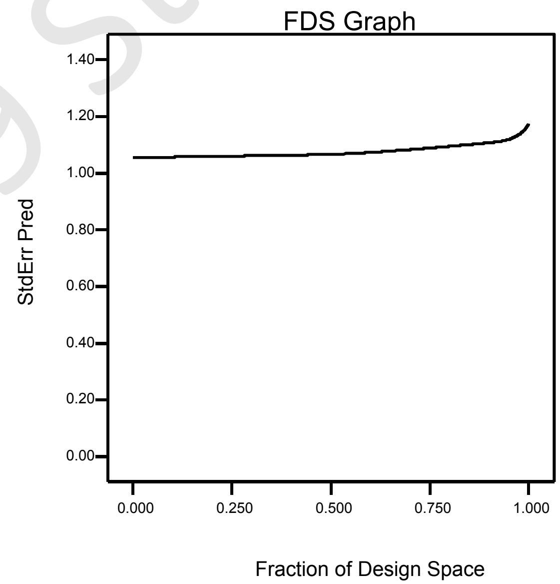
$$StdErr\ Pred\left(FDS\right) = \frac{s_{\hat{y}}}{s} = \sqrt{1 + x_0^T \left(X^T X\right)^{-1} x_0}$$

FDS – StdErr Pred Plot
Fraction of Design Space

1. Pick random points in the design space.
2. Calculate the standard error of a future value

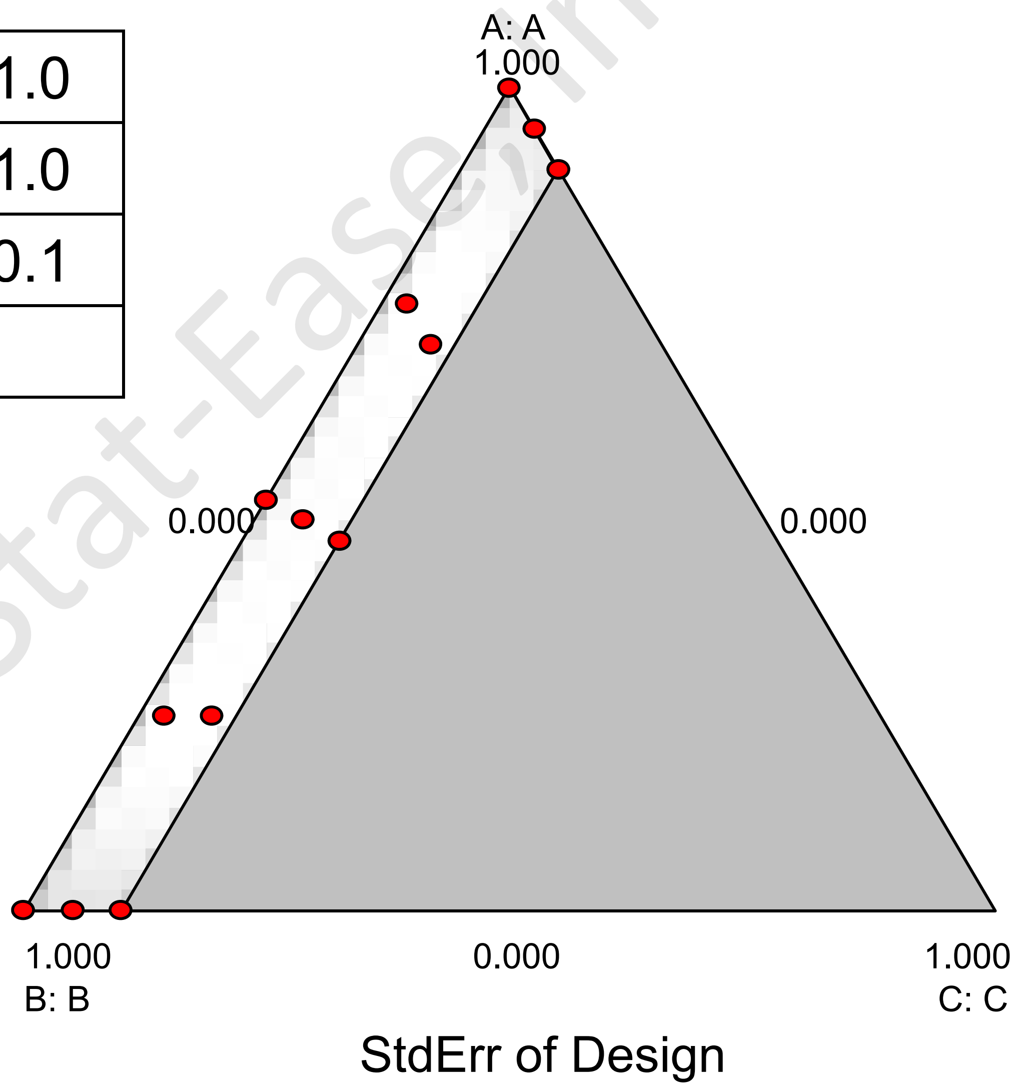
$$\frac{SE_{\hat{y}_0}}{s} = \sqrt{1 + x_0^T \left(X^T X\right)^{-1} x_0}$$

3. Plot the standard error as a fraction of the design space.



Revisit – Mixture Constrained
Design for Quadratic Model

0.0	≤	A	≤	1.0
0.0	≤	B	≤	1.0
0.0	≤	C	≤	0.1
Total = 1.0				



- Want quadratic model to predict a future value within ±18 with 95% confidence.
- The overall standard deviation for this response is 7.8.

For 95% confidence $t_{.05/2,7} = 2.365,\ \delta = 18\ \ \&\ \ s = 7.8$

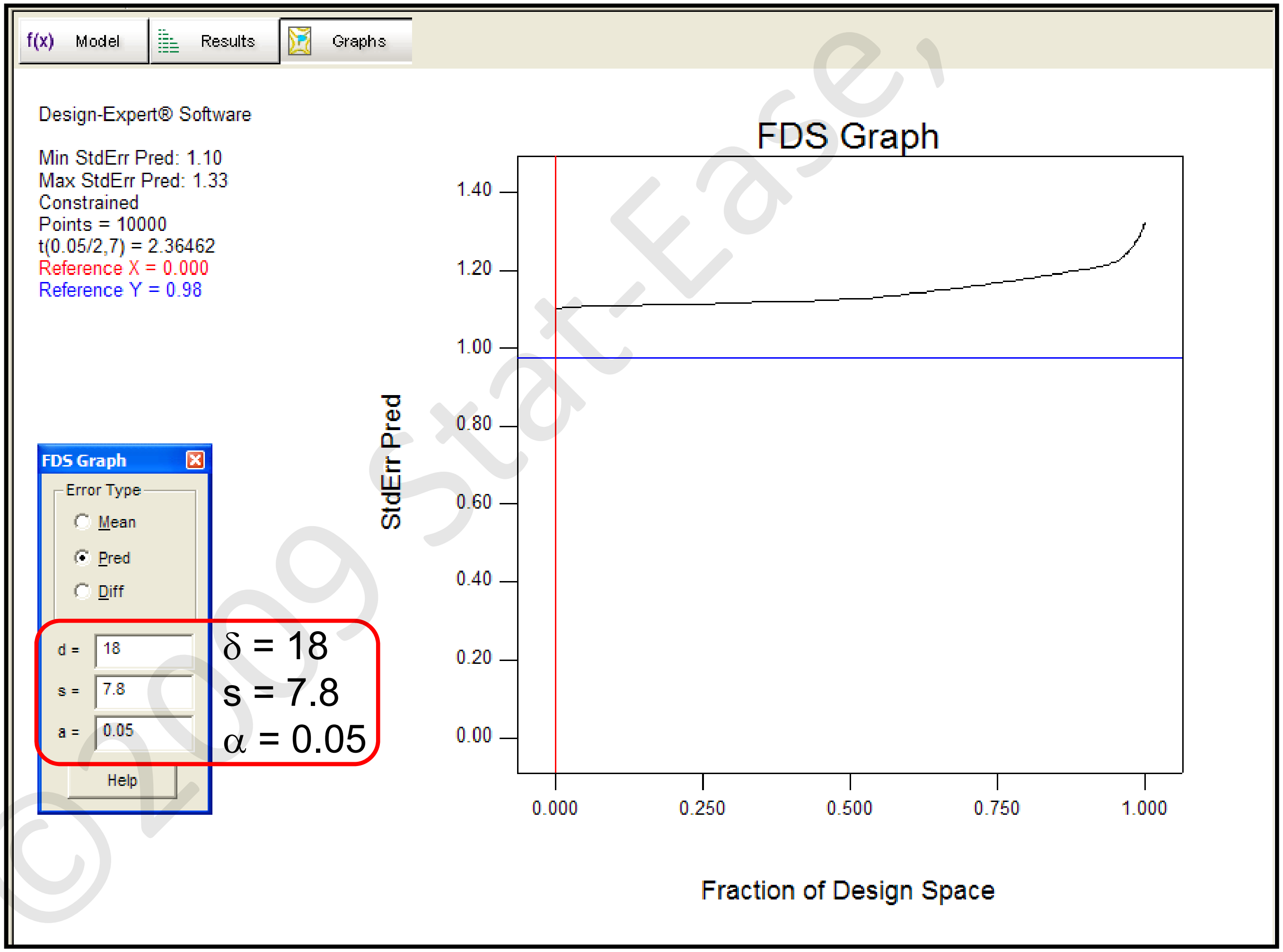
$$s_{\hat{y}} = \frac{\delta}{t_{\alpha/2,df}} = \frac{18}{2.365} = 7.61$$

$$StdErr\ Pred\left(FDS\right) = \frac{s_{\hat{y}}}{s} = \frac{7.61}{7.80} = 0.98$$

Mixture Constrained

Quadratic Model (1 replicate)

0% of the design space has StdErr ≤ 0.98



- Want quadratic model to predict a future value within ±18 with 95% confidence.
- The overall standard deviation for this response is 7.8.

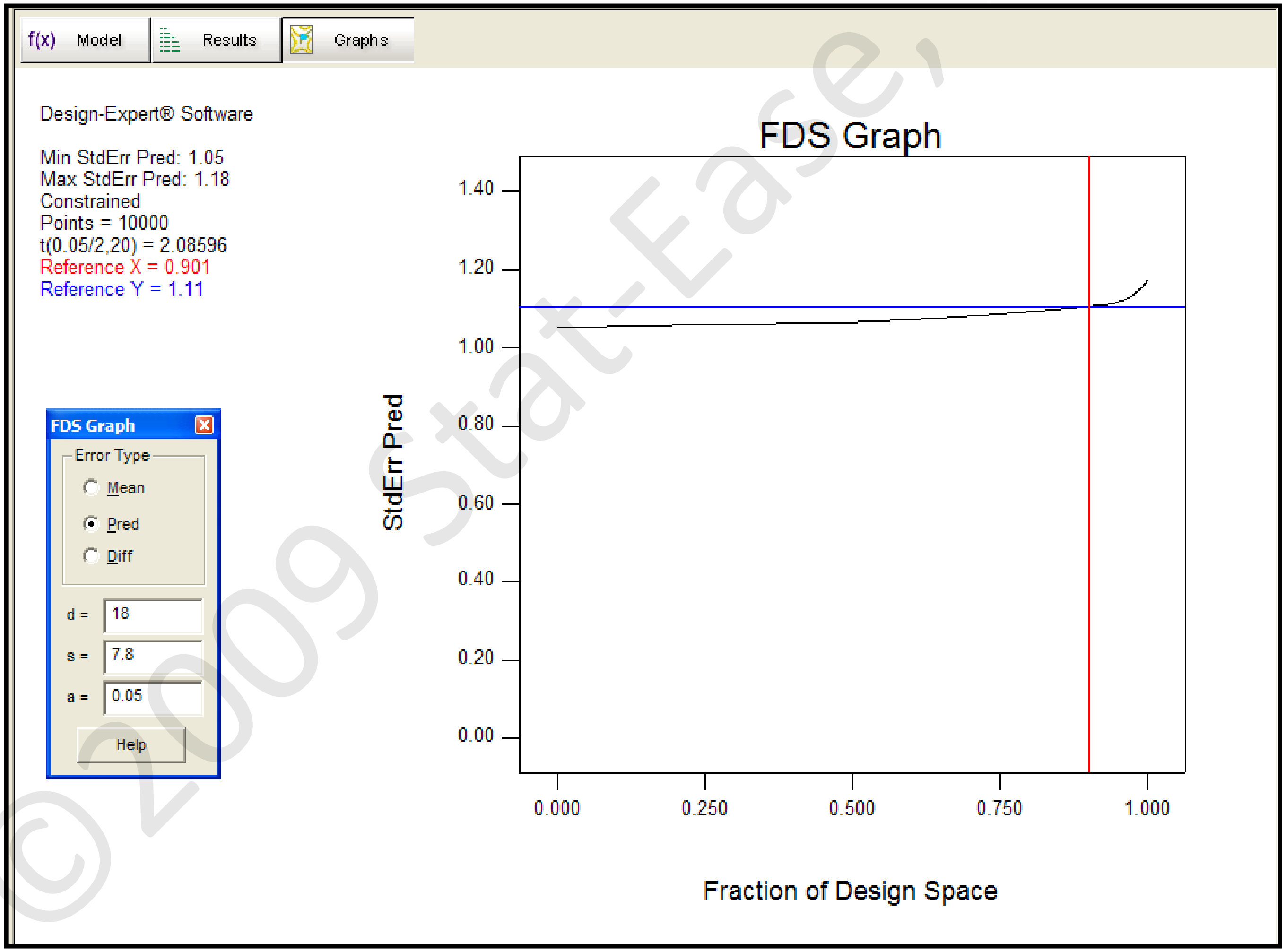
For 95% confidence $t_{.05/2,20} = 2.086$, $\delta = 18$ & $s = 7.8$

$$s_{\hat{y}} = \frac{\delta}{t_{\alpha/2,df}} = \frac{18}{2.086} = 8.63$$

$$StdErr\ Pred\left(FDS\right) = \frac{s_{\hat{y}}}{s} = \frac{8.63}{7.80} = 1.11$$

Mixture Constrained
Quadratic Model (2 replicates)

90% of the design space has StdErr ≤ 1.11



Precision Depends On

- The size of the prediction interval half-width (δ):

  A larger half-width (δ) increases the FDS.

- The size of the experimental error σ:

  A smaller σ increases the FDS.

- The α risk chosen:

  A larger α increases the FDS.

- Choose design appropriate to the problem:

  Size the design for the precision required.

Extracted from MIX

Coating Viscosity
Sizing for Prediction

You sell a coating comprised of resin and two solvents:

- Different applications require different viscosities.
- The customer specifies a viscosity and you formulate a product to that viscosity.
- After much work, you find that instead of making a unique batch to fill each order, you can blend two base resins (a low and high molecular weight) with the two solvents to produce the desired viscosity.
- Now the two base resins can be mass produced and orders filled by blending from bulk storage tanks. (*A huge savings!*)

Extracted from MIX

Coating Viscosity

Build a Design

1. Build a design for a quadratic model (use defaults):

15	≤	A: low MW resin	≤	100
15	≤	B: high MW resin	≤	100
10	≤	C: solvent X	≤	100
10	≤	D: solvent Y	≤	100
Total = 100				
48	≤	A + B	≤	58

2. There is one response: Viscosity in mPaSec

Extracted from MIX

Coating Viscosity

Sizing for Prediction

- Want the quadratic model to predict viscosity within $\pm$ 50 with 95% confidence.
- The overall standard deviation for this response is 20.

For 95% confidence $t_{.05/2,10} = 2.228,\ \delta = 50\ \ \&\ \ s = 20$

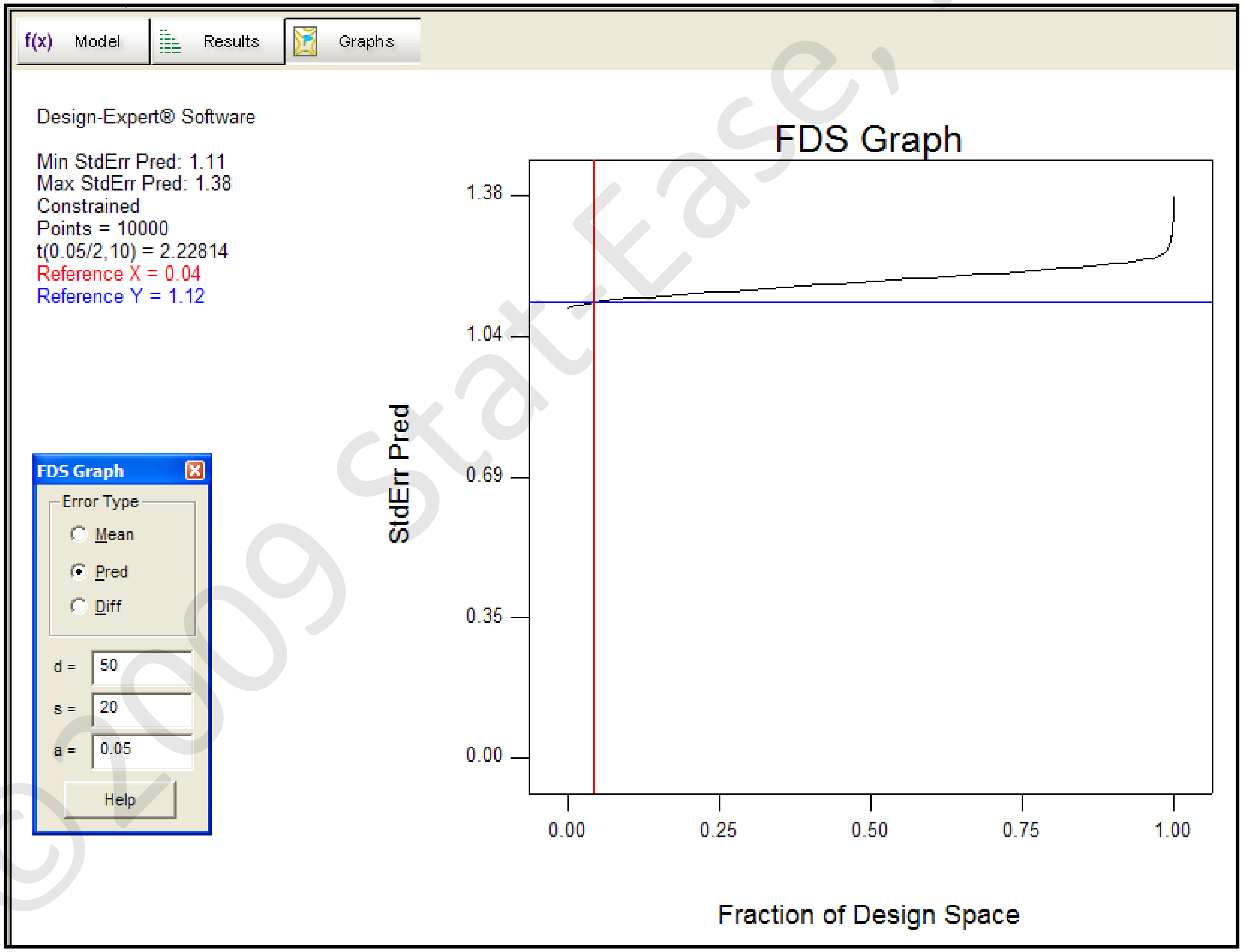
$$s_{\hat{y}} = \frac{\delta}{t_{\alpha/2,df}} = \frac{50}{2.228} = 22.4$$

$$StdErr\ Pred\left(FDS\right) = \frac{s_{\hat{y}}}{s} = \frac{22.4}{20} = 1.12$$

Extracted from MIX

Coating Viscosity
Sizing for Prediction

4% of the design space has StdErr Pred ≤ 1.12



Coating Viscosity
Sizing for Prediction

After pleading for more runs to achieve adequate precision your budget is increased by five runs. Should these runs be added as, model, lack of fit or replicate points?

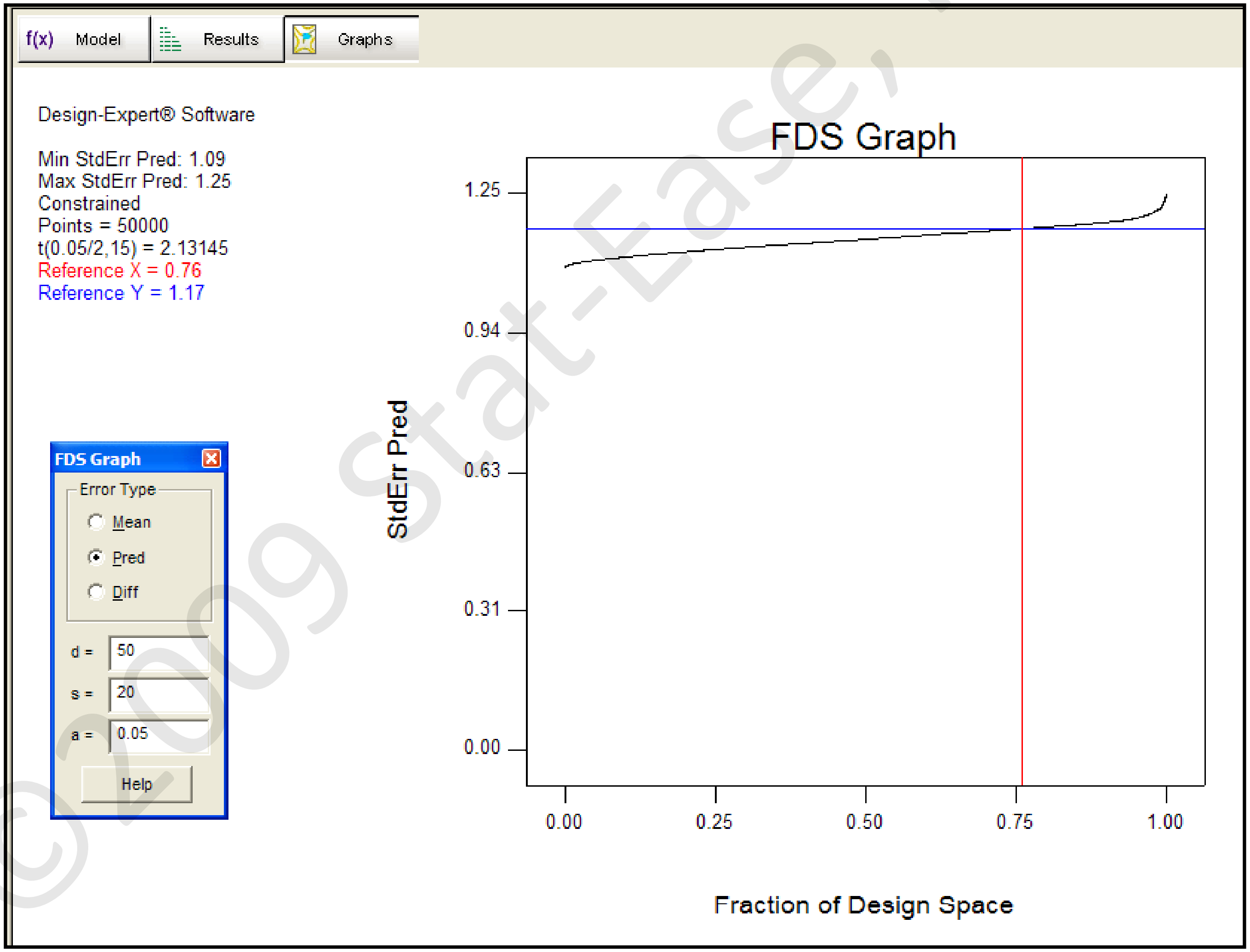
Answer: Model points!

Rebuilt your design (**No** to “Save changes…”; **Yes** to “Use previous design info”) with **15** (rather than the minimum of 10) model points, 5 lack of fit and 5 replicates.

Extracted from MIX

Coating Viscosity
Sizing for Prediction

76% of the design space has StdErr Pred ≤ 1.17



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