A Balancing Act: Optimizing a Product's Properties

Make intelligent trade-offs through desirability optimization methodology.

by
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THE QUALITY COMMUNITY HAS LARGELY ignored a major issue in product improvement: how to improve products from the standpoint of property trade-offs. The problem is one of optimizing property balance. It arises because, as one property is improved, it is often at the expense of one or more other properties.

The trade-off problem is particularly chronic in formulation-related products, such as adhesives, rubber, plastics, inks, paints, and alloys. It is not uncommon to have to balance as many as 20 properties for such formulation products. For example, the abrasion resistance of a rubber shoe sole formulation might be improved by adding certain fillers. But this change might also result in more complex production processes and higher production costs. The problem becomes considerably more complex when several components of the formulation are being varied.

Balance problems can be solved by using a modified formula of E.C. Harrington's desirability function[1] and combining it with response surface methodology (RSM)[2] to form a methodology called desirability optimization methodology (DOM). Computer implementation of DOM further enhances its power.

Why is property balancing difficult?

To see why property balancing can be difficult, consider a rubber formulation with properties of tensile strength, hardness, cost, and rebound. When balancing, difficulties result from deciding how to average properties measured in different units since the different properties cannot be measured using the same scale. If they could be measured using the same scale, it would be possible to simply take an average of the various properties and maximize it—that, however, is precisely the purpose of the desirability function. It provides a metric for balance.

The desirability function ranges between zero and one. Any property can be mapped onto the function. A desirability of zero represents a property level that is expected to render the product unacceptable for use. A desirability of one represents a property level at which a small increase or decrease will not further improve the product.

The following looks at how the desirability function can be used for both discrete and continuous cases.

A discrete case

Figure 1 shows four formulations, each with three properties: tensile, hardness, and elongation. Above the tabular information is one desirability curve for each property. For any property value, a zero-to-one desirability can be found from the corresponding desirability curve. In addition to the desirability curves, each property is associated with a weight. This allows different importance levels to be assigned to different properties.

In this example, the assigned weights imply that hardness is twice as important as tensile and elongation is four times more important than tensile. It must follow that elongation is twice as important as hardness.

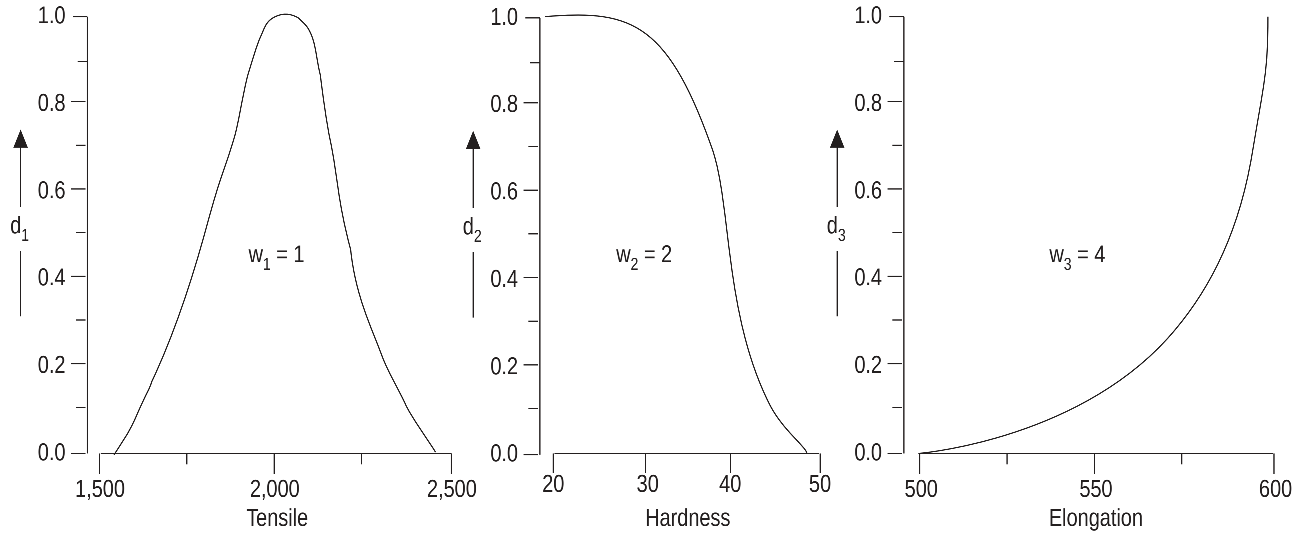
Both the desirability curves and weight assignments are individual or group judgments. (This important factor is discussed in the sidebar "Where Do Desirability Curves and Weights Come From?") Now that each property can be assigned a zero-to-one desirability, the three individual desirabilities for each formulation can be combined into a composite desirability scale, denoted D, that represents average desirability. D, however, is a geometric mean, not an arithmetic mean, because in most product development situations, a single property at an unacceptable level renders the product useless. Even if this is not the case, it is a useful model for product development.

The far right column is the weighted composite desirability, which is calculated from the equation:

$$D = [(d_1)^1(d_2)^2(d_3)^4]^{1/7}$$

Composite desirability takes into account the

Figure 1. Desirability Optimization for the Discrete Case



Candidate formulation	Tensile	d_1	Hardness	d_2	Elongation	d_3	Composite desirability (D)
1	1,750	0.40	45	0.10	550	0.15	0.15
2	2,000	1.00	30	0.97	500	0.00	0.00
3	1,600	0.07	35	0.87	525	0.05	0.12
4	1,600	0.07	30	0.97	585	0.47	0.44

Composite desirability

$$D = \left[d_1^{w_1}\, d_2^{w_2}\, d_3^{w_3}\right]^{\frac{1}{\Sigma w_i}}$$

property weights, which are exponents of the individual desirabilities. To find the best formulation, simply look for the largest D value in the far right column. Clearly, formulation No. 4 represents the best balance of properties. This is a simple example that shows how useful the desirability concept can be when working with a large data base of property values.

A continuous case

By combining the desirability concept with RSM, DOM is created. One can think of DOM in terms of substituting a set of prediction equations (one equation for each property) for the data base in the previous example. These equations have the capability of predicting each pertinent property level for any set of independent variable levels within the area of the data. Therefore, D values can also be produced from these equations, which allows further analysis through the creation of contour surfaces, the location of local maxima/minima, and the use of other techniques.

To illustrate how DOM works, an example with one X variable will first be considered, which will then be expanded to include any number of independent and dependent variables. In Figure 2, plot A is a plot of tensile (Y_2) vs. filler level (X_1). This is an equation fitted to the four data values shown. Plot B translates the tensile scale into a zero-to-one desirability, or d scale. This plot indicates that below 800 pounds per square inch (psi), the product will be unacceptable. At 1,400 psi, a level is reached where higher values are of no added utility. Between 800 and 1,400 psi, higher values correspond to higher values of desirability and thus greater product functionality. Considering plots A and B together, it becomes clear that any filler level within the range of the data will yield a desirability. For example, at 10 parts per 100 parts of rubber (phr), the line shows how one would find d_i for 10 phr filler level. Of course, the same operation can be done for any filler level.

In plots C and D, the same procedure is performed with viscosity, except that, for this property, it is desired to target viscosity at 60 and force values greater than 72 and less than 40 to be zero desirability.

For any filler level (X_1) input, both a d_1 value and a d_2 value can be obtained. Assuming equal importance for the two properties ($w_1 = w_2 = 1$), a plot of composite desirability can be constructed as a function of filler alone, using the equation:

$$D = (d_1 d_2)^{1/2}$$

Composite desirability is shown in plot E. This plot clearly indicates that, for the given desirability curves and weight assignments, the optimum formulation is equal to that of 10 phr

Figure 2. Desirability Optimization for the Continuous Case

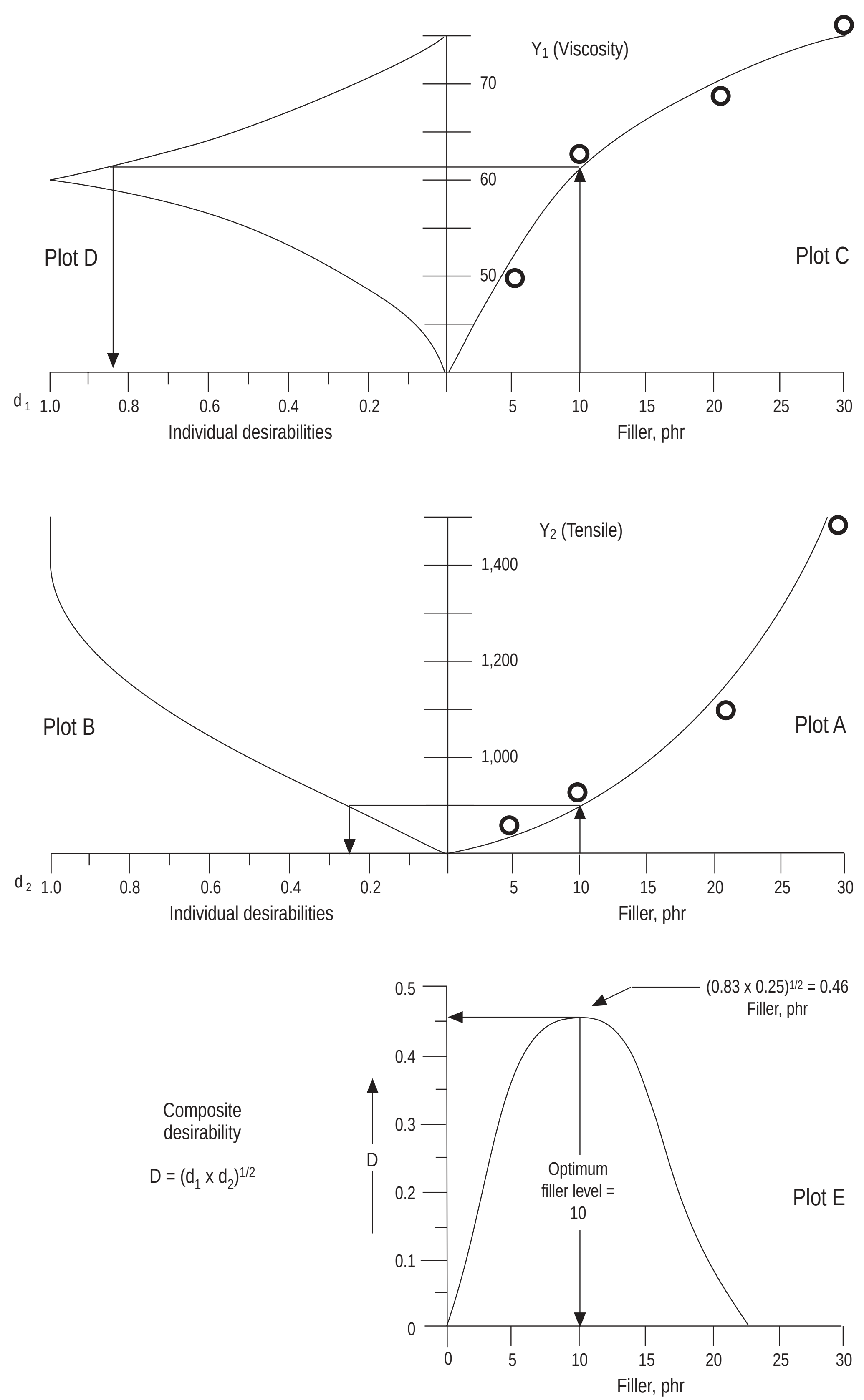


Figure 3. Desirability Optimization for Multiple X's and Y's

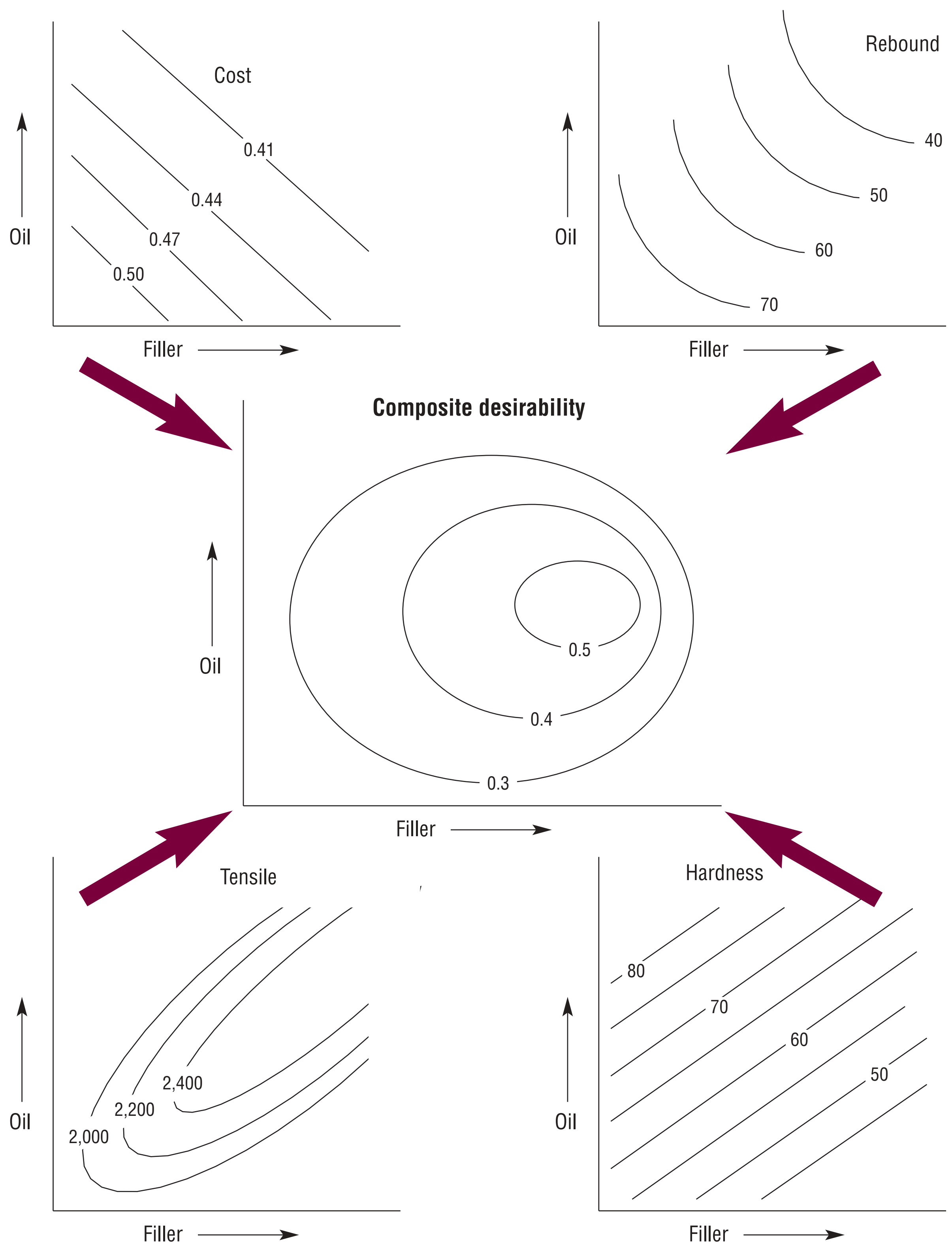
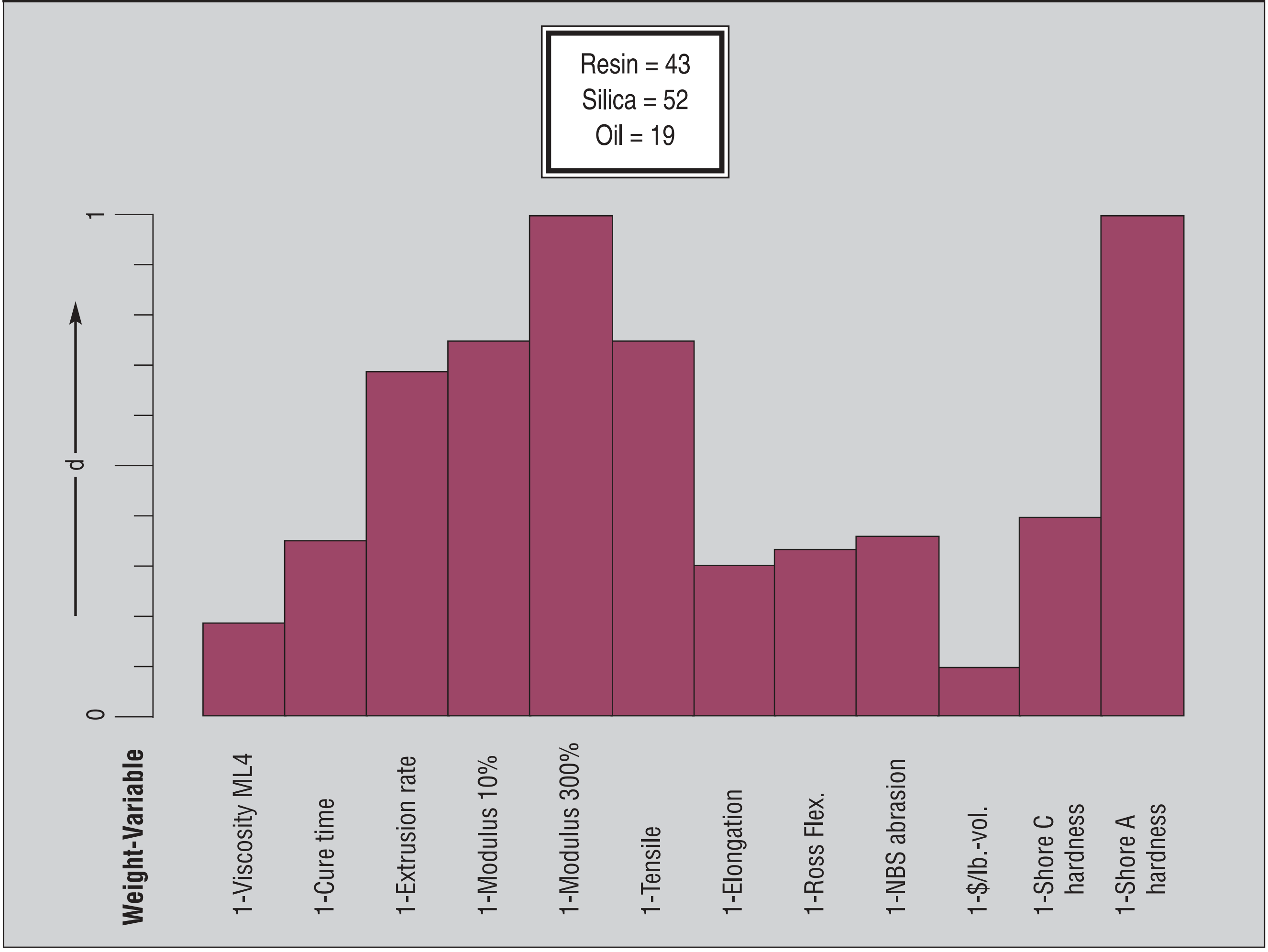


Figure 4. Optimum Formulation and Desirability Histogram for Equal Weight Case



filler. There is no limit to the number of properties that can be stacked up and optimized as is shown in Figure 2.

Although this example has been illustrated graphically for ease of understanding, the procedure can also be expressed in mathematical terms. Y_i is a function of X_1 (filler in this case), d_i is a function of Y_i, and D is a function of d_1 and d_2 . Therefore, D is simply a function of X_1.

Expansion to multiple independent variables

Conceptually, adding independent variables does not create a problem; instead of each dependent variable being a function of one independent variable, X, it becomes a function of two or more X's. All that is needed are data that are appropriate for fitting a prediction equation. Extensive experience in the chemical, physical, and engineering sciences has shown that second-order polynomial equations are flexible enough to fit most behaviors encountered in production or research-and-development practices.

The only remaining problem is what specific combinations of X-variable levels will enable the fitting of such an equation. The RSM discipline has thoroughly addressed this subject.[3] Thus, there are numerous response surface designs from which to choose. RSM is a powerful tool that permits the mathematical modeling of one or more responses as a function of several factors (i.e., independent variables). Therefore, it is easy to explore the consequences of a change in one factor (or a combination of factors) on all responses.

Figure 3 is a schematic for the cases with multiple X's and Y's. The four peripheral contour plots represent properties as a function of two X variables. Each point in the $X_1 - X_2$ space is transformed into a zero-to-one desirability value. The resultant four d values are then combined into a composite desirability, D, which is represented in the center contour plot. The highest point on the center plot corresponds to the optimum formulation. Again, this maximum can be calculated mathematically using a maximization algorithm.

Computer implementation

Although DOM has been used for more than 20 years, its implementation has been tedious and has required a knowledge of (or software for) experimental design, regression analysis, optimization, and graphics. Thus, many have avoided using it. Fortunately, easy-to-use software for DOM implementation is now available.[4] With the availability of such software, added advantages of DOM have come to light, particularly the ability to do what-if simulations and to enhance robust product design. These will be demonstrated using SynGenics Corporation's ImproveIt software.

Figure 5. Optimum Formulation and Desirability Histogram for Optimization With Cost Heavily Weighted

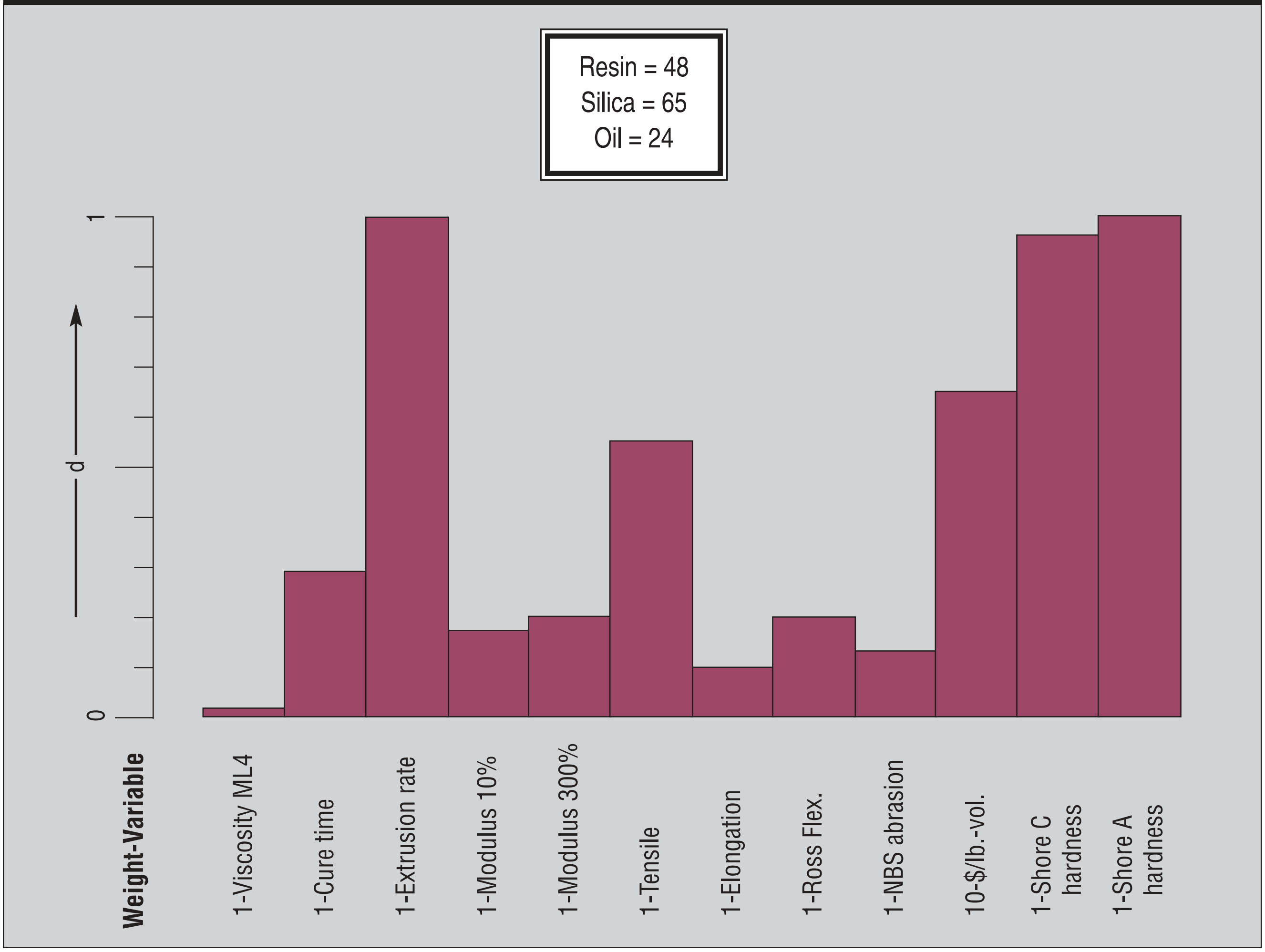
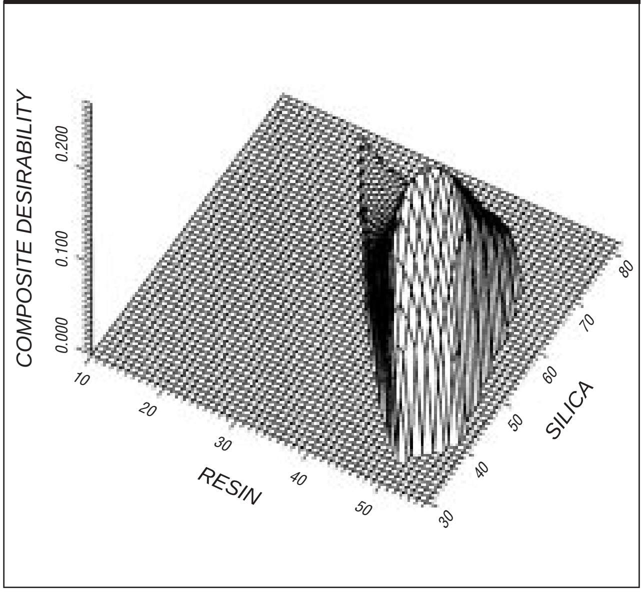


Figure 6. Composite Desirability at Optimum Representing Nonrobust Optimum



What-if scenarios

Figure 4 shows results from the optimization of a rubber shoe sole formulation.[5] It displays the property levels expected from the optimum formulation, which consists of 43 parts of resin, 52 parts of silica, and 19 parts of oil. The bars are plotted on the desirability scale. Note the trade-offs among the various properties, which are represented by the different sized bars. Some desirabilities are high, and some are near zero. It is important to remember that, by definition, if a d value is greater than zero, the formulation is acceptable.

Using this same example, suppose that a price-sensitive formulation needs to be developed. To obtain this, the optimization is simply repeated, with cost being given a higher weight. The resulting property spectrum for this price-sensitive product is shown in Figure 5. As the figure shows, the optimum X-variable settings have changed considerably. To increase the cost desirability (i.e., decrease the cost), other properties had to decrease in desirability, although the resulting formulation is still acceptable. Many other possibilities suggest themselves. For example, the same trade-off analysis can be performed to make the shoe sole wear longer and be more skid resistant on wet surfaces. In this case, the property of abrasion resistance would be given more weight.

Robust product design

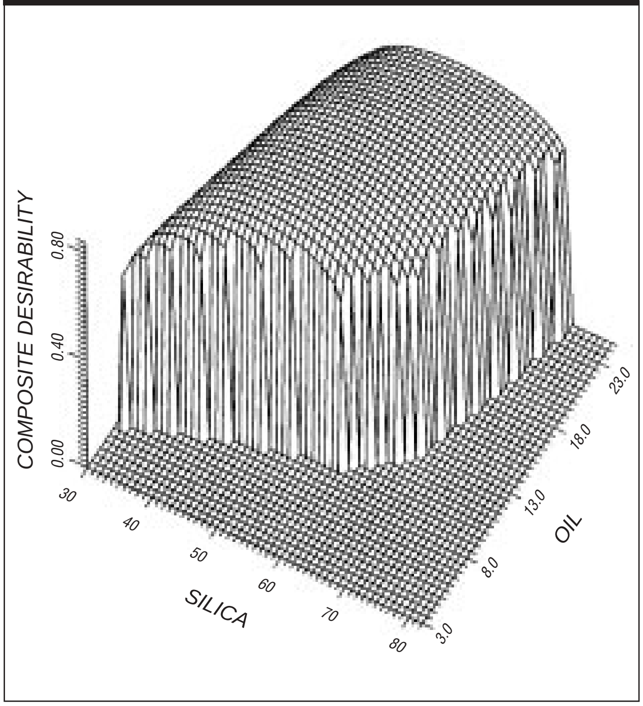
A robust product is one that is resistant to the sources of variability both during product manufacture and product use. Variance reduction has become another phrase that captures the essence of robust product design—that is, making a product resistant to variability.

An optimal formulation product is represented by a maximum D value. In other words, it represents the top of a desirability hill. For more than two X variables, the product is represented by a hilltop in hyperspace. It is important to determine how steep the hilltop is so that product stability and manufacturability can be predicted. A steep peak translates into increased variability in manufactured product; a relatively flat hilltop means that the resulting manufacturing process is much more forgiving.

For example, Figures 6 and 7 represent extreme cases. In Figure 6, it is clear that small perturbations in the level of the X_i at the peak translate to large deviations in D, which in turn translate into large variations in product quality. In Figure 7, the product would be more robust at the optimum D. The gradient at the peak can be expressed mathematically using the gradient function.[6]

This example represents an after-the-fact examination of product robustness. It is also possible and desirable to include a measure of robustness into the desirability function. To do so, the individual response gradients should be included in the composite desirability function.[7]

Figure 7. Composite Desirability at Optimum Representing More Robust Optimum



Where Do Desirability Curves and Weights Come From?

Desirability curves represent engineering judgments on how individual properties or responses translate into product quality. They come either from individual judgments or group consensus.

If the desirabilities are specified by an individual, that person should be very familiar with the product. Ideally, however, the process of assigning desirability curves and their weights is best done by consensus in the early stages of product conception. The consensus meeting should include an expert facilitator and representatives from all functional areas involved with the product, including marketing, sales, production, and in some cases, customer service.

The tangible product of a consensus meeting is a set of agreed-on desirability curves and assigned weights. Consensus ensures that the product development team has a well-defined goal before it spends time and money developing the product.

Participants in desirability facilitation meetings often express satisfaction with the fact that potential design and production problems are identified up front before a lot of expense is incurred fixing them later in the process. Here is an enlightening example: In one meeting involving two different laboratories in two different countries, the facilitation session revealed that one lab had spent two years trying to maximize a particularly important property while the other had been busy trying to minimize it. Although an extreme example, it underscores the importance of thinking through the process and product before rushing into development and production.

The desirability optimization methodology is most effective when implemented after a facilitation session is carried out. It provides an ideal structure for obtaining the consensus judgments necessary for developing world-class products.

A proven methodology

DOM is a proven methodology for developing products in which many properties must be balanced. It is especially useful when those properties must be balanced against each other.

Acknowledgment

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References

1. The desirability function was introduced by E.C. Harrington Jr. in "The Desirability Function," *Industrial Quality Control*, April 1965, pp. 494-498. It was modified by G.C. Derringer and R. Suich in "Simultaneous Optimization of Several Response Variables," *Journal of Quality Technology*, October 1980, pp. 214-219.

2. For more information on RSM, see G.E.P. Box, W.G. Hunter, and J.S. Hunter, *Statistics for Experimenters* (New York, NY: John Wiley and Sons, 1981); G.E.P. Box and N.R. Draper, *Empirical Model Building and Response Surfaces* (New York, NY: John Wiley and Sons, 1987); and G.C. Derringer, "Response Surface Methodology" in *Applied Techniques in Statistics for Selected Industries*, edited by H.E. Hill and J.W. Prane (New York, NY: John Wiley and Sons, 1984), pp. 425-459.

3. Ibid.

4. The available software packages for DOM implementation on a personal computer (PC) include ImproveIt by SynGenics Corporation (25 W. New England Ave., Worthington, OH 43085) and PROOPT by L.R. Good and Son (7053 Morse Rd., Alexandria, OH 43001). A sta-

tistical PC program that has added DOM is Design-Expert® Version 3 by Stat-Ease, Inc. (Hennepin Square, Ste. 480, 2021 E. Hennepin Ave., Minneapolis, MN 55413). MINITAB Inc. plans to add DOM capability to its software by summer 1994 (3081 Enterprise Dr., State College, PA 16801). The author has been told that other software packages, including Macintosh programs, are also available, but their names are unknown at this time.

5. Example courtesy of PPG Industries, Inc.

6. E.J. Buckeler and I.M. Kristensen, *Journal of IRI*, January/February 1967, pp. 28-34.

7. G.C. Derringer, "Use of Response Surface Methodology to Increase Product Robustness," Proceedings of Quality Concepts 1993 Conference, Warren, MI, Oct. 4-6, 1993, pp. 213-231.

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