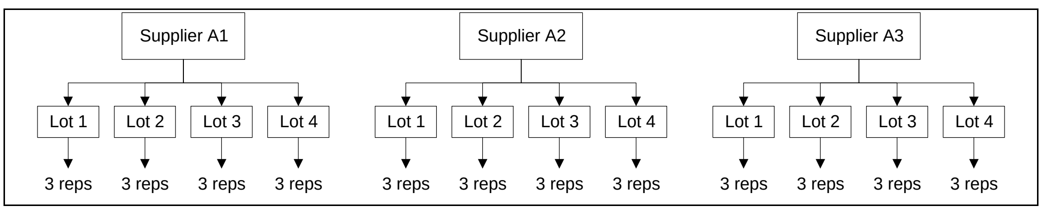
Nested General Factorial Tutorial

Introduction

In some experiments the levels of one factor (e.g. factor B) are similar but not identical for different levels of another factor (e.g. factor A). This arrangement is call a "nested" or "hierarchical" design. The analysis of a nested design is tricky, even for statisticians. It can be done on Design-Ease® software by properly designating effects in specific ways for subsequent analysis of variance. Proceed with care!

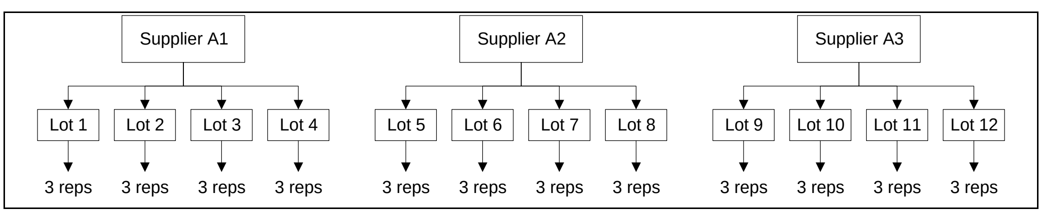
To illustrate how Design-Ease software can be used for a nested design, let's do an example from Montgomery's *Design and Analysis of Experiments*. A company buys raw material from three different suppliers. They want to know if raw material purity is dependent on the supplier. Four lots of raw material are selected at random from each of the suppliers. Three independent measures of purity are made on each batch of raw material.

As shown on the diagram below, this is a nested design.



Flowchart of nested design on raw material supplier

The lots of raw material are nested within supplier. Lot 1 under supplier A1 is not the same lot as lot 1 under supplier A2 or A3. Another (perhaps more correct) diagram of this design is shown below. Note that the lots are unique.



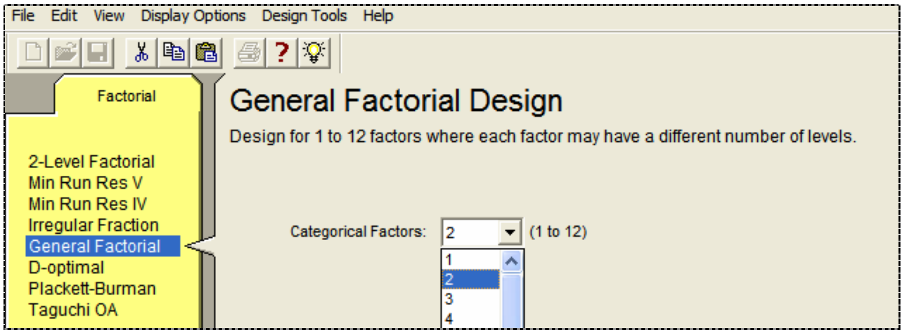
A more realistic way to view the lot-by-lot variation in nested design

Here we can clearly see that the lot is dependent on supplier, that is, raw material lots are nested within supplier. Another name for a nested design is a "hierarchical" design. Think of the lots as children and supplier as a set of parents. Each lot is uniquely tied to its supplier, just as a child is to its parents.

Nested designs are a complex topic and this tutorial is intended to demonstrate only the mechanics. For theory see Montgomery's textbook.

Design the Experiment

Click the blank-sheet icon on the toolbar (or choose File, New Design). Then from the default Factorial tab click **General Factorial**. Enter **2** as the number of factors or select from the downlist as shown.

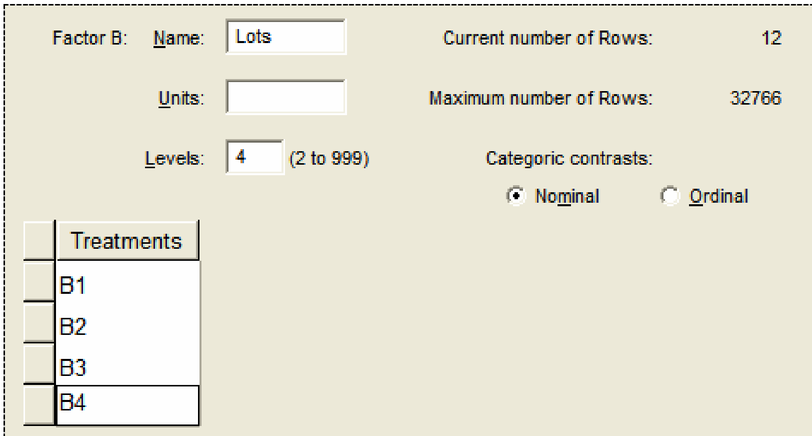


Setting up a two-factor general factorial design

Click on the **Continue** button and enter as **Name** for factor A **Supplier** and change **Levels** to **3**. Enter for the **Treatments** the names **A1**, **A2** and **A3**. Your screen should now look like the illustration.

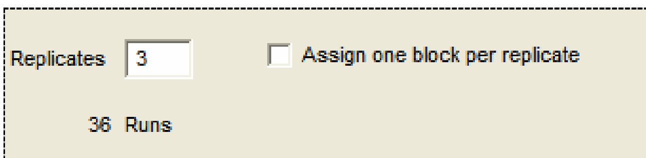
Entering first factor (A)

Click **Continue** to move on to the next factor. Enter for its name **Lots** and **4** for the number of levels. Change the treatments to **B1**, **B2**, **B3** and **B4**.



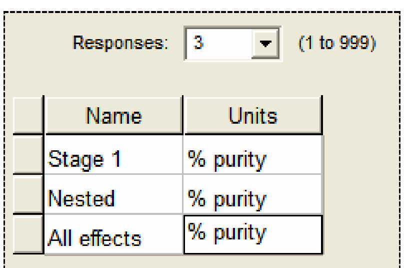
Details on the second factor (B)

Continue to the next screen and enter for **Replicates** the number **3** (for the three independent measures of purity). Press the **Tab** key to update the number of runs.



Specifying three replicates

Click on the **Continue** button. Change the number of responses to **3**. Enter the **Name** as **Stage 1** (for assessing effect of supplier), **Nested** (for analysis of lot-to-lot variation) and **All effects** (will use this for graphs). For the **Units** enter **% purity** using copy (Ctrl-C) and paste (Ctrl-V) to save on keystrokes if you are a poor typist.



Response names

Click on the **Continue** button. Design-Ease now displays, in random run order by supplier and lot, the samples to test for purity.

Analyze the Results

To avoid the trouble of typing in data, simple read in the responses via the icon or via File, Open Design from the main menu. Select the file named **Purity.de7**. You should now see the data depicted in the table below. The purity values are based on a nominal value of 93 percent. For example, a value of 4 indicates a purity of 97 (93 + 4).

Presumably this rescaling of the response made it easier for people to interpret the results.

Run	A: Supplier	B: Lots	% purity
1	A1	B1	-1
2	A3	B2	2
3	A2	B2	0
4	A1	B3	0
5	A3	B3	2
6	A1	B3	-2
7	A2	B1	-3
8	A3	B4	3
9	A3	B2	-2
10	A2	B3	-1
11	A3	B4	2
12	A1	B2	-4
13	A3	B3	-1
14	A1	B4	0
15	A2	B3	-2
16	A3	B3	1
17	A1	B4	1
18	A3	B1	0
19	A2	B2	2
20	A2	B3	0
21	A1	B3	1
22	A3	B1	4
23	A2	B2	4
24	A1	B1	1
25	A1	B2	-2
26	A3	B1	2
27	A2	B1	1
28	A1	B2	-3
29	A1	B1	0
30	A1	B4	4
31	A3	B4	1
32	A2	B4	0
33	A3	B2	0
34	A2	B1	-2
35	A2	B4	3
36	A2	B4	2

Response data for tensile strength of paper

Since this is a nested design, the analysis is not as straight-forward as usual for Design-Ease software. You must create separate ANOVA's for the top-level (stage 1) treatment (supplier – a fixed factor). Then you fit the full model in order to get meaningful diagnostics and model graphs (but you ignore the full model ANOVA!). To keep these various analyses straight, the purity results have been copied three times over.

Statistical Details

Feel free to skip past these details and resume the tutorial at the analysis of variance (ANOVA) phase for stage 1.

In order to perform the correct F-test, you must determine which terms belong in the model and in the error. (Remember that $F=MS_{model}/MS_{error}$.) For our example, B is a random factor (a sample from a population of lots) that is nested within A, which is a fixed factor (there are only three suppliers of interest). The expected mean squares (EMS) are shown in the table below. (For more statistical details, see Montgomery's 6th edition of *Design and Analysis of Experiments* – chapters 13 ("Experiments with Random Factors") and 14 ("Nested and Split-Plot Designs").)

Source	df	Expected MS	Calculated MS
A - Supplier	2	$\sigma^2 + 3\sigma^2_B + 6\Sigma A^2$	$(SS_A)/(df_A)$
B - Lots	9	$\sigma^2 + 3\sigma^2_B$	$(SS_B + SS_{AB})/(df_B + df_{AB})$
Error	24	σ^2	$MS_{Pure\ Error}$
Total	35		

Expected mean squares

From the expected MS column it can be inferred that A (Supplier) should be tested against the nested factor, B (Lots). The appropriate test for significance of A is then:

$$F = (\sigma^2 + 3\sigma^2_B + 6\Sigma A^2)/(\sigma^2 + 3\sigma^2_B).$$

From the calculated MS column it can be seen that the correct sum-of-squares for B is created by adding the sum-of-squares for factor B and the AB interaction. You must add these sums of squares manually using the procedure outlined below. Since lots are nested within supplier, there can be no true supplier by lot interaction (AB). Therefore the sum of squares (SS) is:

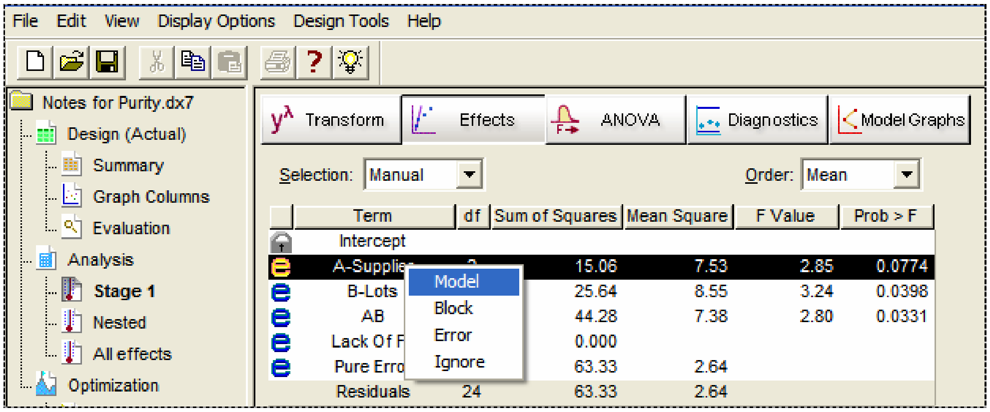
$$SS_{(B\ within\ A)} = SS_B + SS_{AB}.$$

To analyze factor A (supplier), click on the analysis node labeled Purity, which can be found in the tree structure along the left of the main window. Then, click on the Effects button displayed in the toolbar at the top of the main window. There are four states an effect can have:

- Model ("M")
- Block ("b")
- Error ("e")
- Ignore ("X").

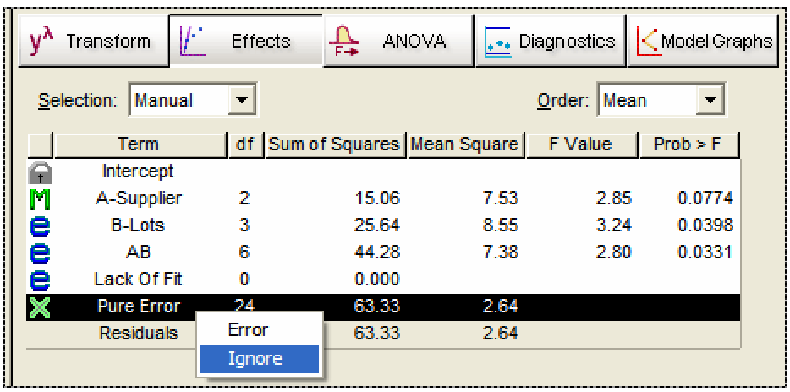
Stage One ANOVA (Supplier)

Supplier (A) is tested against the lots within supplier (B + AB). To perform this analysis, click the analysis node **Stage 1** and press ahead to ***Effects***. Then right-click on factor **A** and choose **Model**.



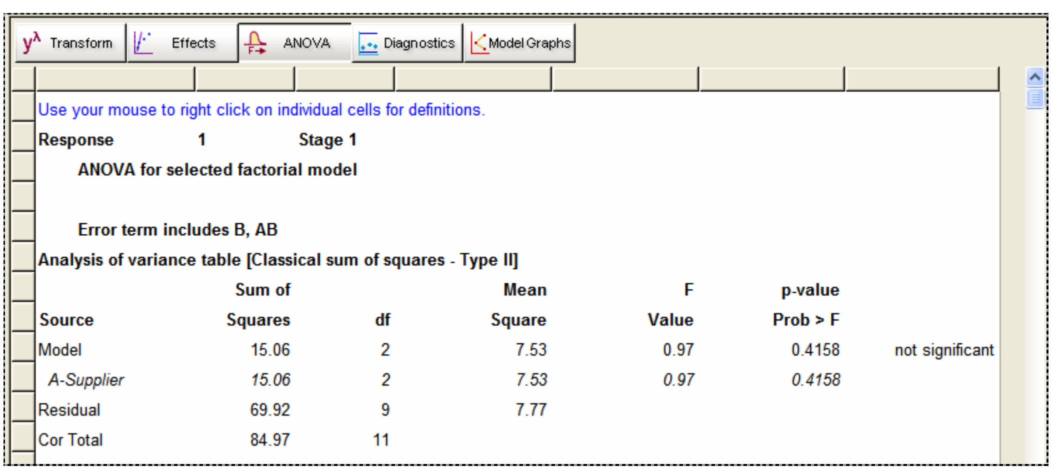
Designating the effect of A for model

<u>Right click</u> on term **Pure Error** and choose **Ignore**. Leave **B**, **AB** and **Lack Of Fit** set as **e** for error (the default). Your screen should now match that shown below.



Completed effects screen in preparation for ANOVA on effect of A

Click on the ***ANOVA*** button. You can see that the supplier (factor A) is not significant (p-value for Prob > F is greater than 0.10).

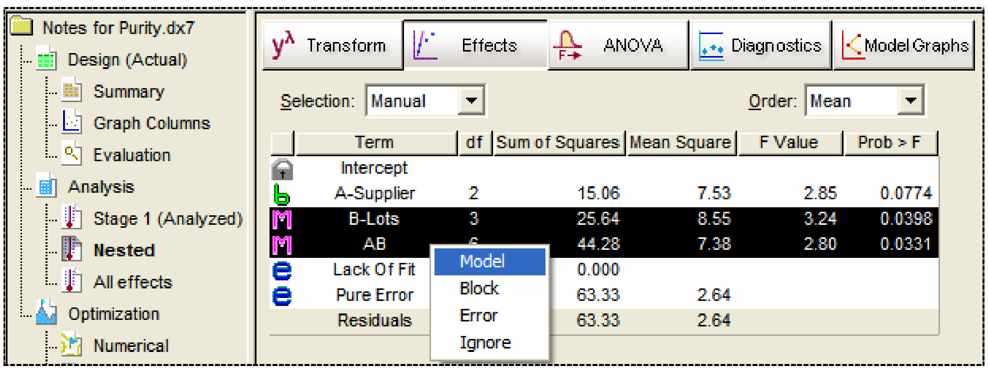


ANOVA for effect of A

Don't bother going any further with analysis because the model is incomplete at this point. However, to preserve your ANOVA on the stage 1 factor of supplier, select **File**, **Save As** and modify the name to **Purity-a.dx7** (or anything else you'd like that will leave the original tutorial file as-is).

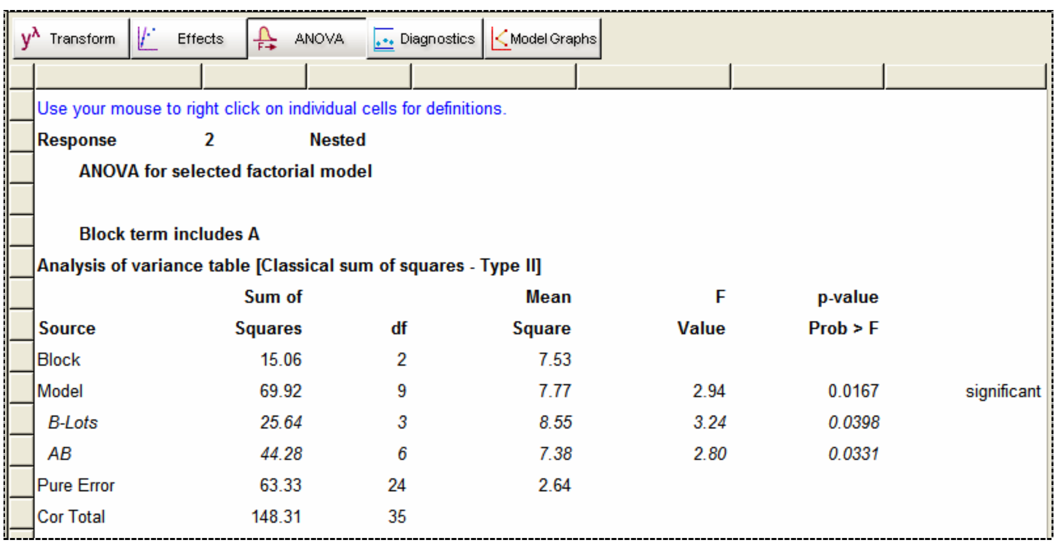
ANOVA on Nested Factor (Lots)

To analyze the variance caused by lots, click the node named **Nested** and then ***Effects***. The lots within supplier should be tested against the pure error. Via the right-click menu, set this up as follows: **A** as **Block**, and **B** and **AB** as **Model**. Leave the **Pure Error** term as **e** for error. Your screen should now match the illustration below. (Notice that by dragging over multiple terms, they all can be designated the same with one operation.)



Designating effects for ANOVA on nested variable (lots)

Click on the ***ANOVA*** button. The results show that lots within supplier (the Model line in the ANOVA) created a significant effect on purity.



Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Block	15.06	2	7.53			
Model	69.92	9	7.77	2.94	0.0167	significant
B-Lots	*25.64*	*3*	*8.55*	*3.24*	*0.0398*	
AB	*44.28*	*6*	*7.38*	*2.80*	*0.0331*	
Pure Error	63.33	24	2.64			
Cor Total	148.31	35				

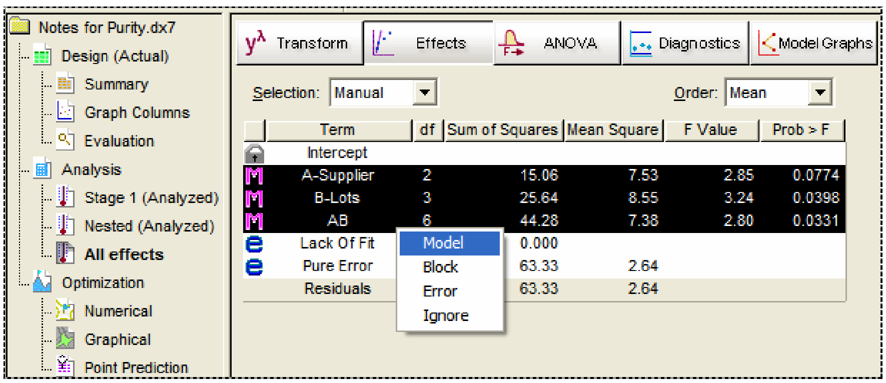
ANOVA for lots within supplier

Do not look at the Diagnostics or the Model Graphs because the model is incomplete at this point. However, to preserve your ANOVA, select **File**, **Save** or simply click the save icon .

Viewing Diagnostic and Model Graphs

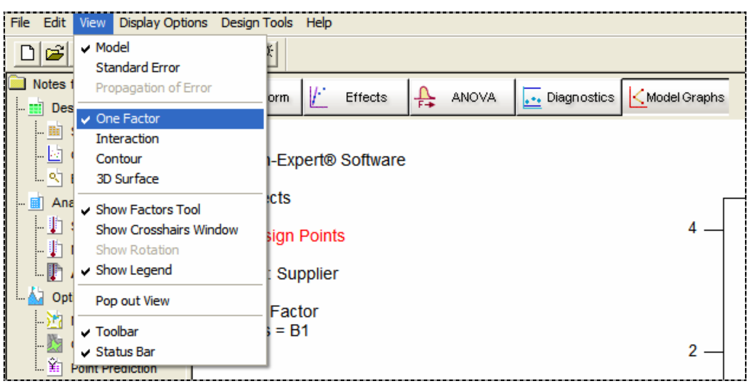
To get meaningful diagnostics and model graphs you need to fit the full model. Of course you must then ignore the ANOVA, because the estimate of error will be incorrect.

Click the **All effects** node under the Analysis branch of your Design-Ease software. Go to the ***Effects***, do a mouse-drag over **A**, **B**, and **AB** and then, via the right-click menu, set these terms to **Model**. Leave **Lack of Fit** and **Pure Error** at **e** for error. Your screen should now match that illustrated.



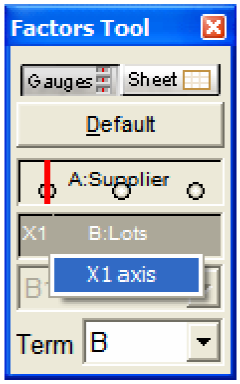
Modeling all the effects

Skip by the ANOVA button (not correct for this complete model) to the ***Diagnostics***. Examine the normal plot of residuals and other graphs available on the Diagnostics Tool. They look good, so click on the ***Model Graphs*** button. From the main menu bar choose **View**, **One Factor**.



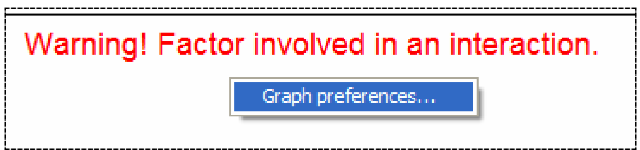
Viewing the main effects via the One Factor model-graph

The default graph shows the effect of supplier on purity, but since it was shown earlier to be statistically insignificant, do not dwell on it. Instead, go to the **Factors Tool**, right click over the **Lots** factor, (which is significant) and make it the **X1 axis**.



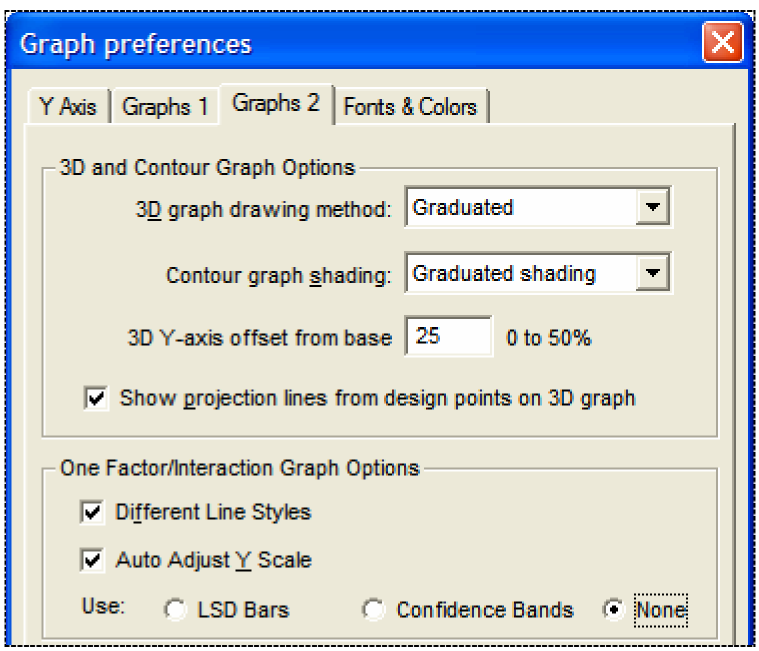
Changing the axis

The least-significant-difference (LSD) bars on this graph will be incorrect due to the nested design for the experiment, so they should be turned off before publication. Do this by a right-click over the plot and selecting **Graph preferences**. (Ignore the warning shown on the graph about the interaction, which in this case will be irrelevant.)



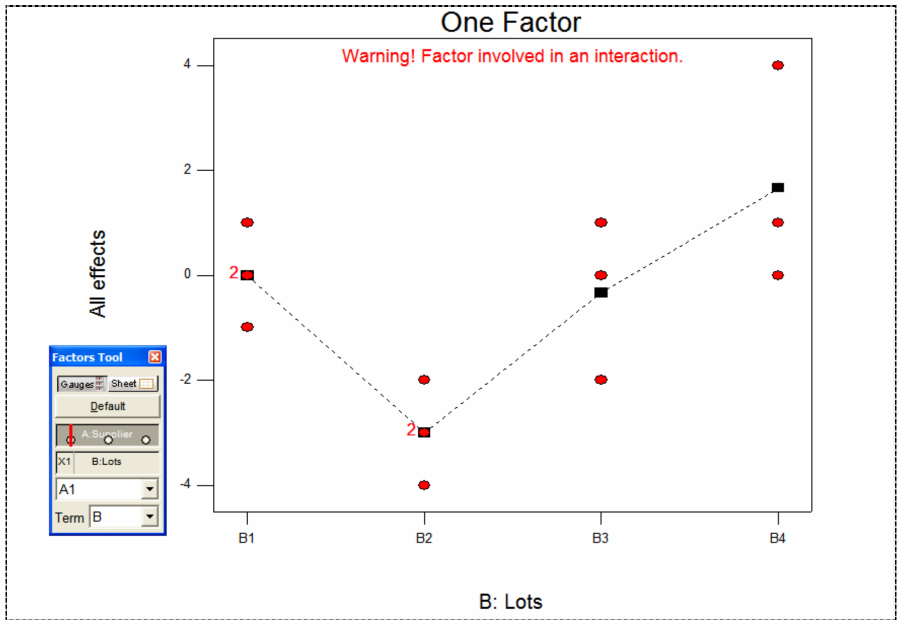
Graph preferences

Click the **Graphs 2** tab and the option **None** (as opposed to LSD Bars).



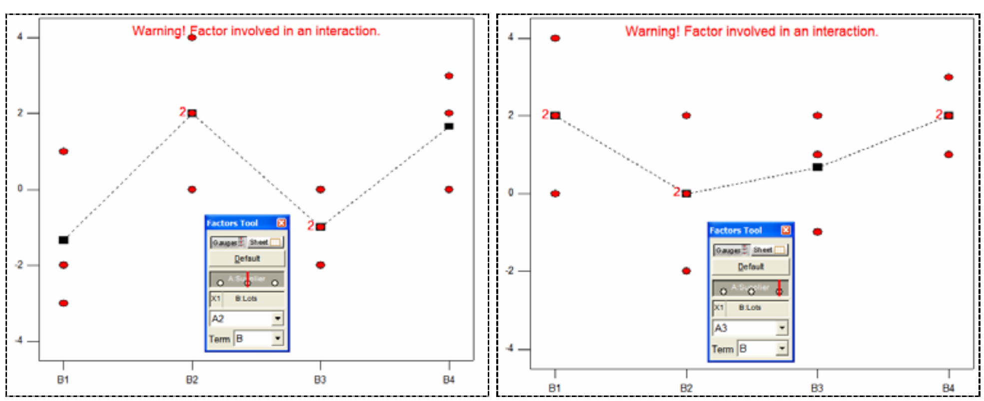
Turning off the LSD bars

OK this change (remember to reverse it back to LSD bars in future when analyzing completely randomized designs). You should now see the graph of lot purity for the first supplier.



Effect graph of lot-to-lot variation from the first supplier

The number 2's shown for lots B1 and B2 indicate that there's an actual point (round symbol) hidden by the mean results (square symbols). Click on the other two buttons for A:Supplier to see how lots vary within suppliers A2 and A3. The patterns change in random fashion, but the amount of variation is considerable.



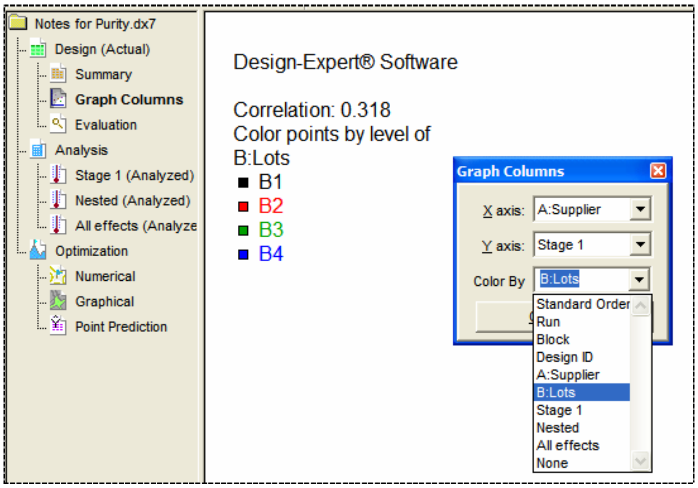
Lot-by-lot variation for the two other suppliers

To preserve the modeling used to produce the plot of lot-by-lot effects, select **File**, **Save** or click the save icon 🖫.

Montgomery summarizes this case study by saying:
"*The practical implications of this experiment and the analysis are very important. The objective of the experimenter is to find the source of the variability in raw material purity. If it results from differences among suppliers, then we may be able to solve the*

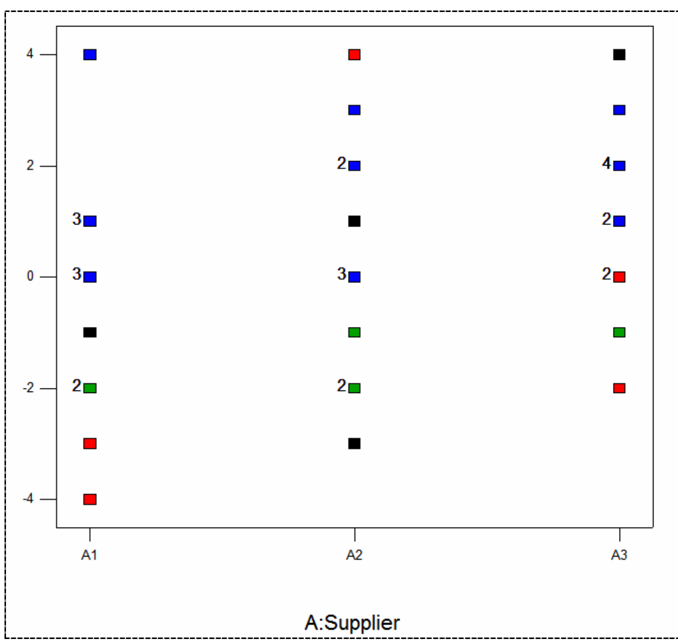
problem by selecting the "best" supplier. However, that solution is not applicable here because the major source of variability is the batch-to-batch purity variation within suppliers. Therefore, we must attack the problem by working with the suppliers to reduce their batch-to-batch variability."

To provide support for Montgomery's conclusions, click on the **Graph Columns** node under the Design branch of your Design-Ease software. Then on the Graph Columns tool click the down list ▼ for **Color By** and select **B: Lots**.



Setting up a graph of purity by supplier, colored by lots

The correlation of purity versus supplier is positive in the direction of supplier A3, but only very weakly. There's too much lot-to-lot scatter within each supplier to see statistically significant differences between them.



Scatterplot of actual purity results

This is the end of the story, but as a postscript, you may wonder if it was worth all the bother to properly account for the nested nature of this experiment. Click back to the ANOVA for the “All effects” analysis and then back to the ones for “Stage 1” and “Nested”. The difference for the supplier effect is dramatic – only by properly accounting for variances does one see that, due to variation by lots, the impact of this factor cannot be reliably estimated from this experiment. In other words, in this particular case, an experimenter who unwittingly ignored the restrictions in randomization caused by the nesting may have unfairly picked on a particular supplier who exhibited lower than normal purity.

Notice that for this particular design structure, the “All effects” ANOVA does provide an accurate test on the nested factor (B – Lots). However, it is best that you not count on this always being the case for nested designs in general.