General Factorial Tutorial (Part 1 – Categoric Treatment)

Introduction – A Case Study on Battery Life

Design-Ease® software version 7 offers a "General Factorial" option on the "Factorial" tab. If you completed the General One Factor Tutorial (recommended), you've seen how this option handles one multilevel, categorical factor. In this tutorial you will learn how to set up a design on multiple categorical factors. Part two shows you how to convert truly continuous factors, such as temperature, from categorical to numerical. Then you can generate response surface graphs that provide a better perspective of your system.

The experiment in this case, which comes from Montgomery's *Design and Analysis of Experiments,* seeks consistently long life in a battery that will be subjected to extremes in ambient conditions. It evaluates three materials (factor A) at three levels of temperature (factor B). Four batteries are tested at each of the nine two-factor combinations in a completely randomized design. The responses from the resulting 36 runs can be seen below.

Material Type	Temperature (deg F)					
	15		70		125	
A1	130	155	34	40	20	70
	74	180	80	75	82	58
A2	150	188	136	122	25	70
	159	126	106	115	58	45
A3	138	110	174	120	96	104
	168	160	150	139	82	60

General factorial on battery (response is life in hours)

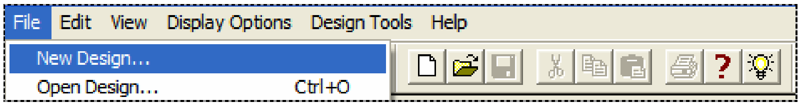
The following questions must be answered:

- How does material type and temperature affect battery life?
- Which material, if any, will give uniformly long life regardless of temperature?

The second question, if it can be answered in the affirmative, leads to the big payoff: a battery that will be robust to temperature variation in the field. This case study provides a good example of the application of statistical DOE for robust product design.

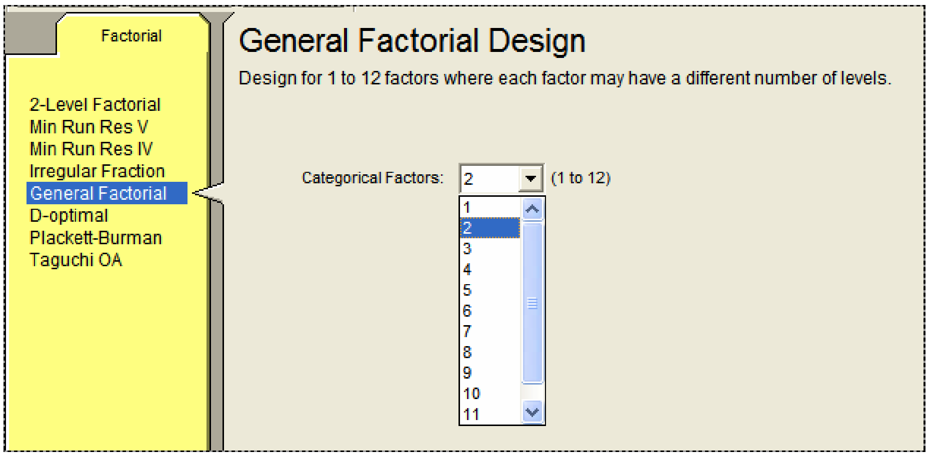
Design the Experiment

To build the design, choose **File**, **New Design** (or to save strokes, simply click the blank-sheet icon (🗋**)** on the toolbar).



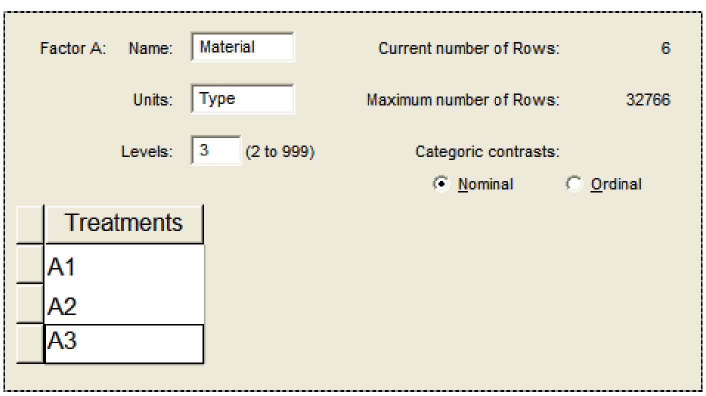
Starting a new design via menu (option: click blank-sheet icon () on the toolbar)

Then from the default **Factorial** tab, click **General Factorial**. Enter **2** as the number of factors.



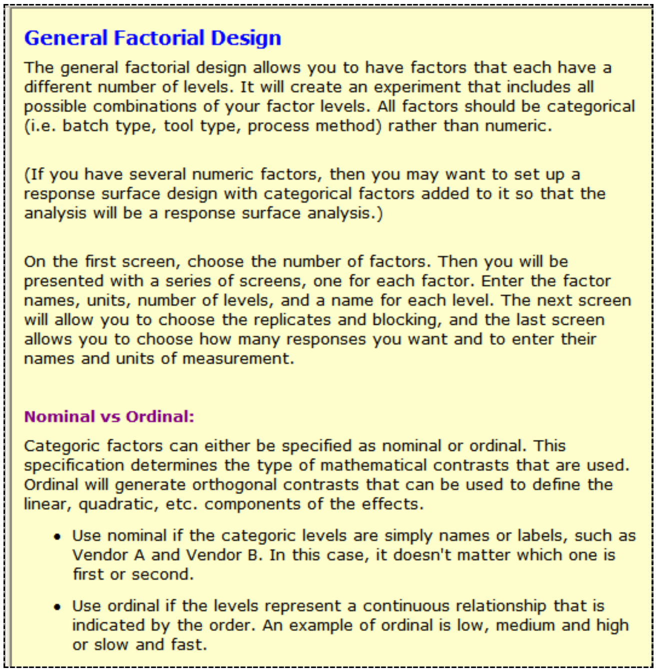
Selecting number of factors for general factorial design

Click the **Continue** button and enter **Material** for the factor name, **Type** for units and **3** for the number of levels as shown in the illustration below. Change the treatment names to **A1**, **A2** and **A3**. Accept the default for this categorical factor to be contrasted in nominal (named) fashion, as opposed to being ordinal (ordered). This difference in the nature of factors affects how Design-Ease codes the categorical levels, which changes the model coefficients reported under ANOVA in the subsequent response analysis.



Entering material as a nominal factor

Tutorials such as this one on general factorials will quickly get you up to speed on how to use Design-Ease software, but it does not serve as a statistical primer for design and analysis of experiments. If you crave such details, Help is at your finger tips! For example, go to **Help**, **Contents** and work your way down the tree structure through the factorial branches to **General (Multi-Level) Factorial Design**. Note the details on the distinction in categoric contrasts (Nominal vs Ordinal).



General Factorial Design

The general factorial design allows you to have factors that each have a different number of levels. It will create an experiment that includes all possible combinations of your factor levels. All factors should be categorical (i.e. batch type, tool type, process method) rather than numeric.

(If you have several numeric factors, then you may want to set up a response surface design with categorical factors added to it so that the analysis will be a response surface analysis.)

On the first screen, choose the number of factors. Then you will be presented with a series of screens, one for each factor. Enter the factor names, units, number of levels, and a name for each level. The next screen will allow you to choose the replicates and blocking, and the last screen allows you to choose how many responses you want and to enter their names and units of measurement.

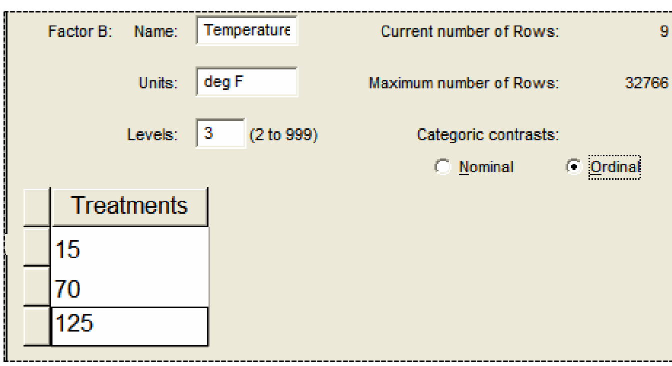
Nominal vs Ordinal:

Categoric factors can either be specified as nominal or ordinal. This specification determines the type of mathematical contrasts that are used. Ordinal will generate orthogonal contrasts that can be used to define the linear, quadratic, etc. components of the effects.

- Use nominal if the categoric levels are simply names or labels, such as Vendor A and Vendor B. In this case, it doesn't matter which one is first or second.
- Use ordinal if the levels represent a continuous relationship that is indicated by the order. An example of ordinal is low, medium and high or slow and fast.

Help on general factorial design

Close Help by pressing **X** on the window and then Continue to entry for factor B. Enter: **Temperature** for the factor name, **deg F** for units, **3** for the number of levels, and **15**, **70** and **125** for the levels. Then press the **Categoric Contrasts** selection for **Ordinal** to recognize that although it's being treated categorically (for example due to controls offering only the three levels), temperature is really a continuous factor.



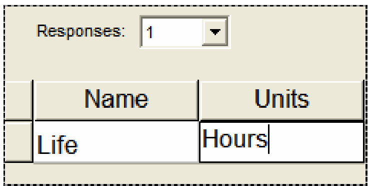
Entering information on factor B

Notice that the current number of rows (9 = 3 x 3) is far less than the maximum. However, you may hit this limit if you include too many factors at too many levels.

Click on the **Continue** button to complete the factor-entry stage of the design build. Enter **4** for replicates. The number of runs won't be updated until you press the Tab key or move from the cell. (Recall from the table of data that four batteries were tested at every combination of material type and temperature.) Leave the blocks option alone, because these experiments are completely randomized.

Entering the number of replicates

Click **Continue** button to move on to the entry screen for responses. Leave the default responses at 1. Enter the name as **Life**, the units as **hours**.

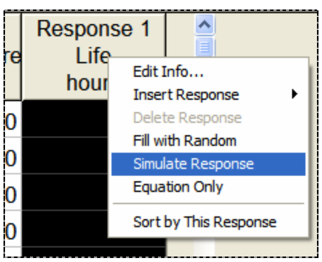


Response entry screen

Click **Continue** to complete the design specification process. Design-Ease now displays the 36 runs (in random order) from the 3x3 factorial design with four replicates.

Analyze the Results

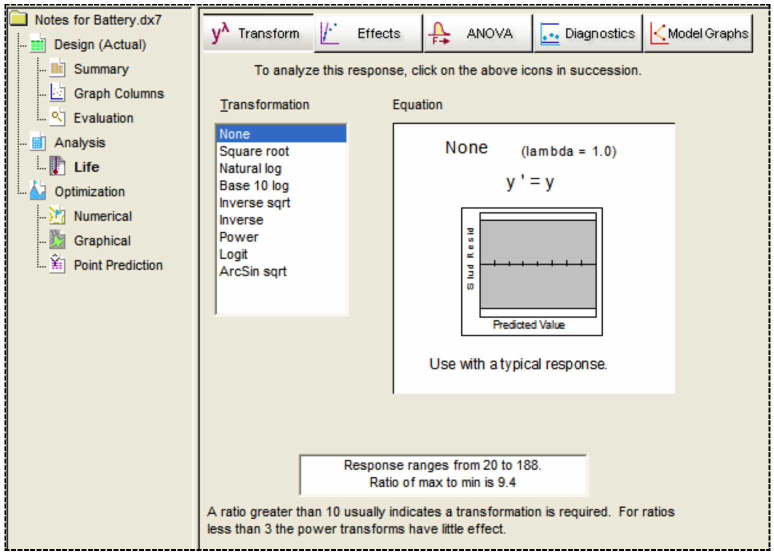
To save time, simulate the experimental results by right-clicking the response header and selecting **Simulate Response**.



Choosing a simulation

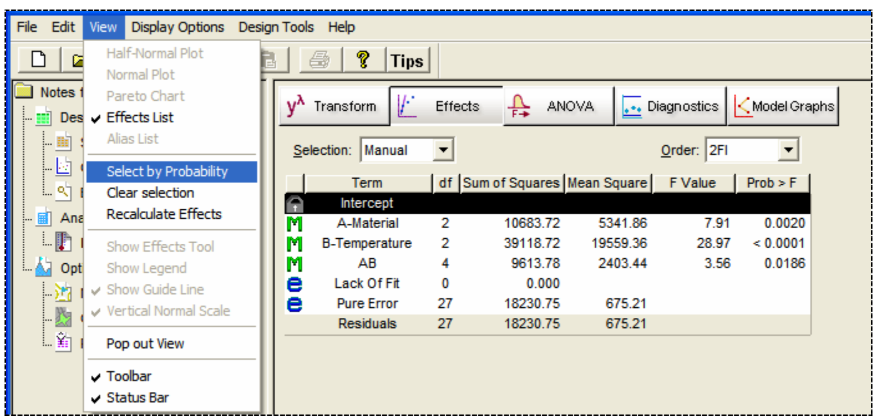
Click the file named **Battery.sim** and **Open** it. You should now see data slowly flow in from the experiment (we added a delay in the simulator so you can read the results as they get entered by the computer – also, this makes it seem a bit harder to do the runs: Let's not make things look too easy!). This is a good time to preserve your work: Select **File** and **Save As**. Change the file name to **Battery.de7** and **Save**.

Then under the **Analysis** branch of the program click the node labeled **Life**. You now see options for performing response transformations. (Note: As discussed in previous tutorials, this screen shot and others refer to the "dx7" equivalent from the more comprehensive Design-Expert program, but your screen shows "de7" as the file type.)



First step in the analysis – transformation options

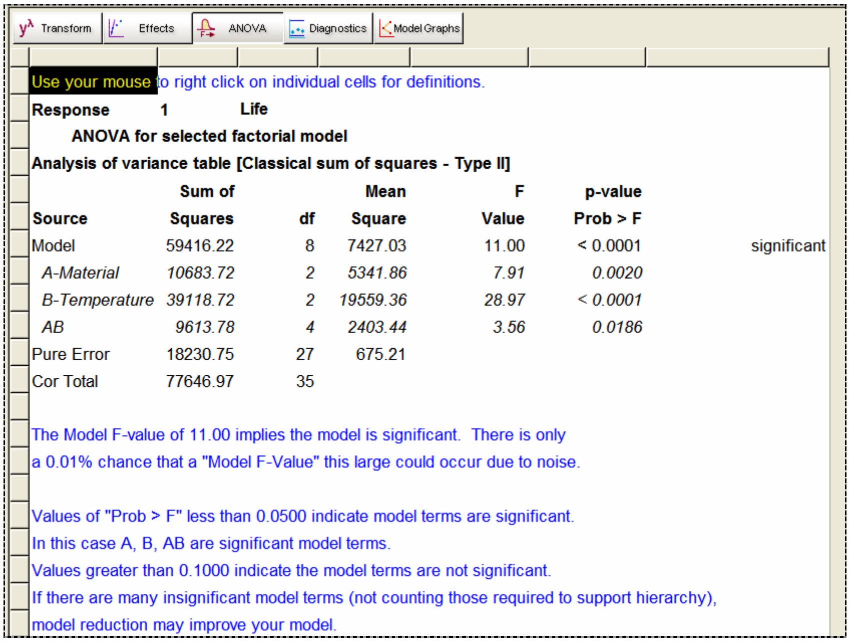
Leaving the transformation at the default of "None", go ahead and click on the ***Effects*** button displayed next in the toolbar for response analysis. Notice that all model terms are designated "e" for error. However, by replicating each of the nine factor combinations four times, 27 degrees of freedom (df) are generated for pure error, which can be used by the program as a benchmark for fitting model coefficients. Do this by going to the main menu item **View** and choosing **Select by Probability**.



Effects View (after Select by Probability)

Notice that the program now selects significant model terms (designated "M") – those whose Prob>F fall below a p-value of 0.05. (Note: The default statistical criteria for selection can be configured by going to Edit, Preferences for Math.)

Click the ***ANOVA*** button to see the complete analysis of variance. If you do not see annotations, select View, Annotated ANOVA.



Use your mouse to right click on individual cells for definitions.

Response 1 Life

ANOVA for selected factorial model

Analysis of variance table [Classical sum of squares - Type II]

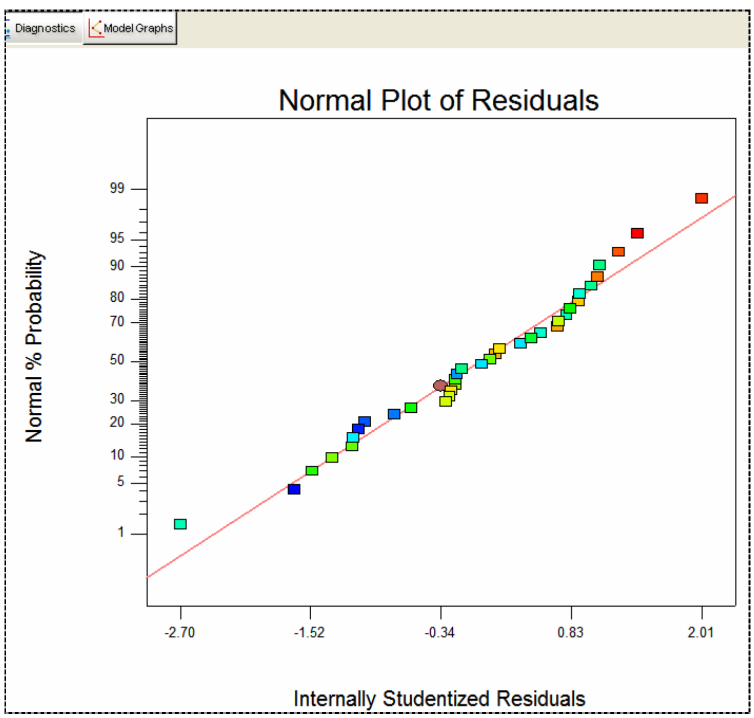
Source	Sum of Squares	df	Mean Square	F Value	p-value Prob > F	
Model	59416.22	8	7427.03	11.00	< 0.0001	significant
A-Material	*10683.72*	*2*	*5341.86*	*7.91*	*0.0020*	
B-Temperature	*39118.72*	*2*	*19559.36*	*28.97*	*< 0.0001*	
AB	*9613.78*	*4*	*2403.44*	*3.56*	*0.0186*	
Pure Error	18230.75	27	675.21			
Cor Total	77646.97	35				

The Model F-value of 11.00 implies the model is significant. There is only
a 0.01% chance that a "Model F-Value" this large could occur due to noise.

Values of "Prob > F" less than 0.0500 indicate model terms are significant.
In this case A, B, AB are significant model terms.
Values greater than 0.1000 indicate the model terms are not significant.
If there are many insignificant model terms (not counting those required to support hierarchy),
model reduction may improve your model.

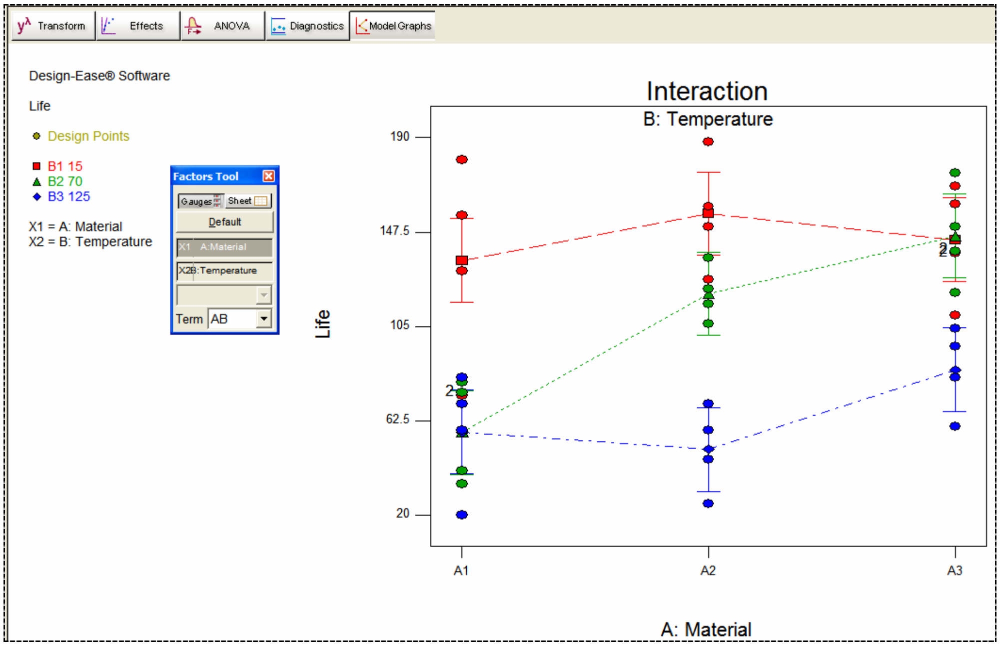
Annotated ANOVA Report

Scroll down to see post-ANOVA statistics such as R-Squared. As you can conclude for yourself by reading the comments, the results look good. Further down the report are details on the model based on nominal contrasts. We provide a breakdown on this in the Experiment Design Made Easy workshop. To keep this tutorial moving, it's best not to get bogged down in the mathematics of modeling categorical factors, so press ahead to the ***Diagnostics*** button and examine the residual graphs.



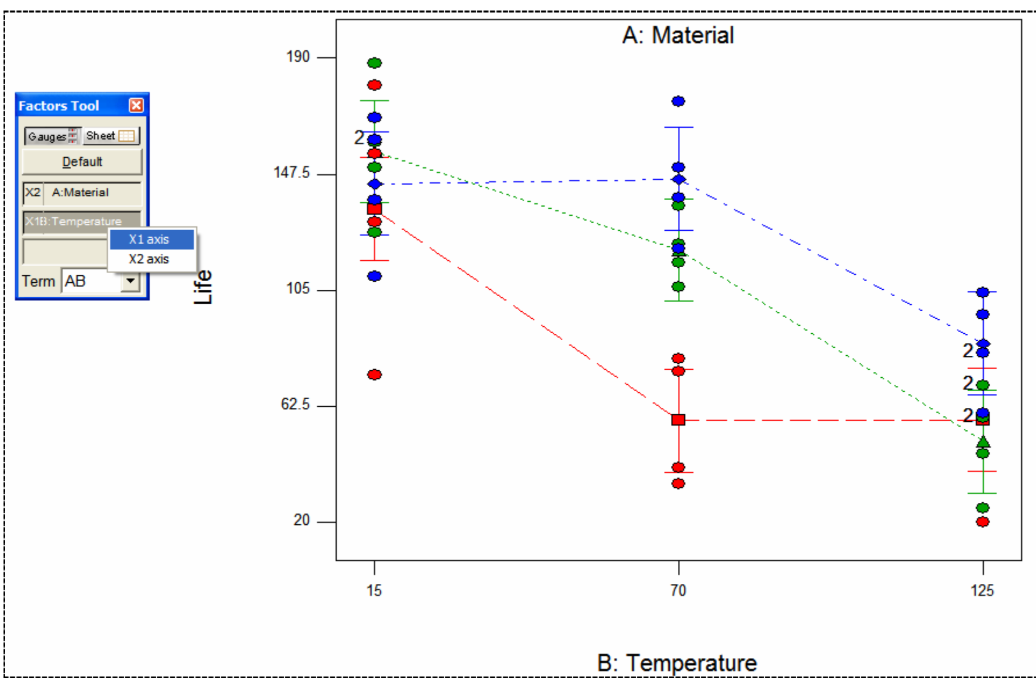
Normal plot of residuals – looks OK

The diagnostics produce nothing abnormal so click the ***Model Graphs*** to view the results.



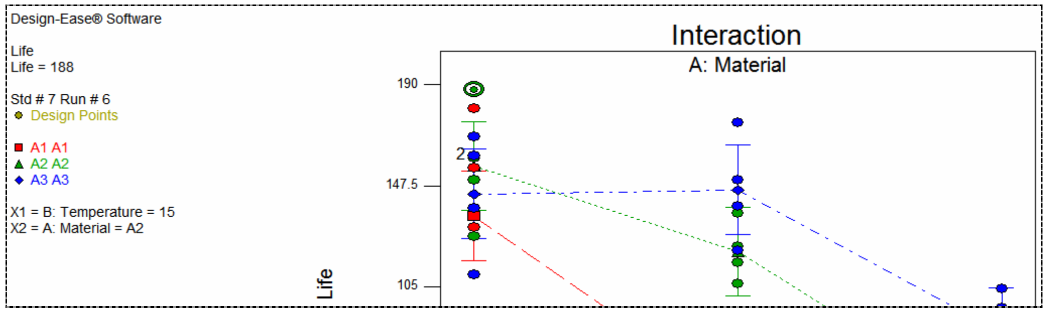
Default model graph – interaction plot with A on bottom (X1) axis

Right click over the **Temperature** factor on the floating graph tool and change it to the **X1 Axis**, thus producing an interaction graph with the ordinal factor displayed in a continuous manner and the nominal factor (material) laid out discretely as separate lines. This will make it easier to interpret the results.



Effect graph with temperature on bottom axis

To see how the software identifies points, click on the highest one (green) at the upper left in the graph.



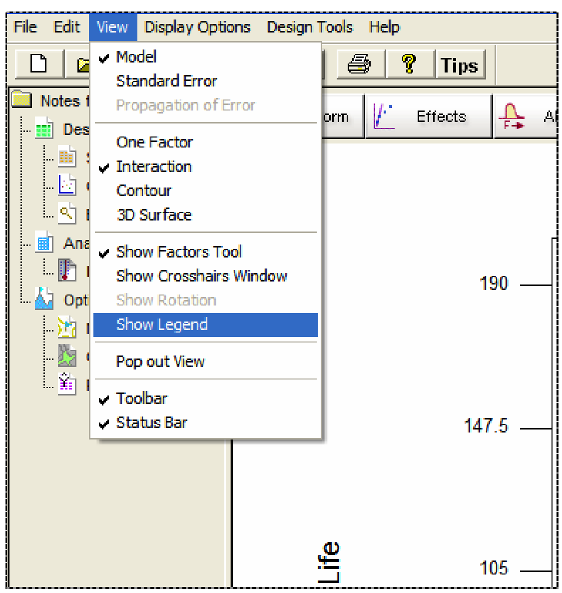
Point highlighted for identification

Note how to the left of the plot the software identifies the point by:

- the actual result (188)
- standard order number (7)
- run number (due to randomization yours may differ from that shown)
- factor levels (temperature of 15 with material A2).

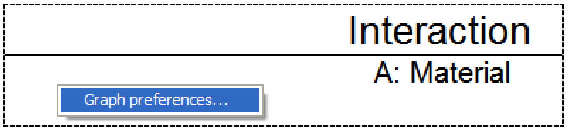
The actual results are represented by various-colored circles. You can also click on the non-circular symbols (square, triangle or diamond) to display the predicted outcome and least significant difference (LSD). Try this!

To produce a cleaner looking plot, go to **View** and deselect **Show Legend**.



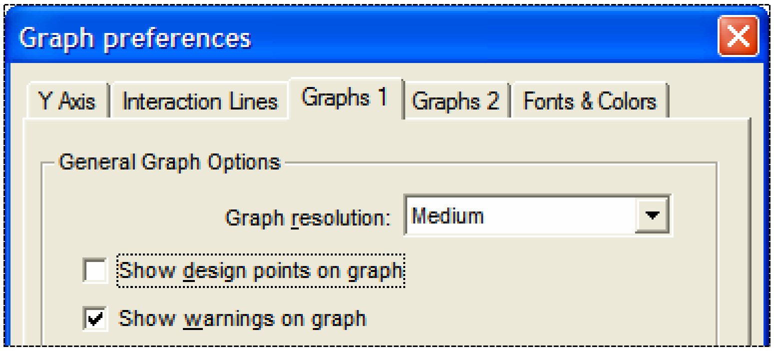
Legend turned off

Let's do some more clean-up for report purposes: <u>Right-click over the graph</u> and select **Graph Preferences**.



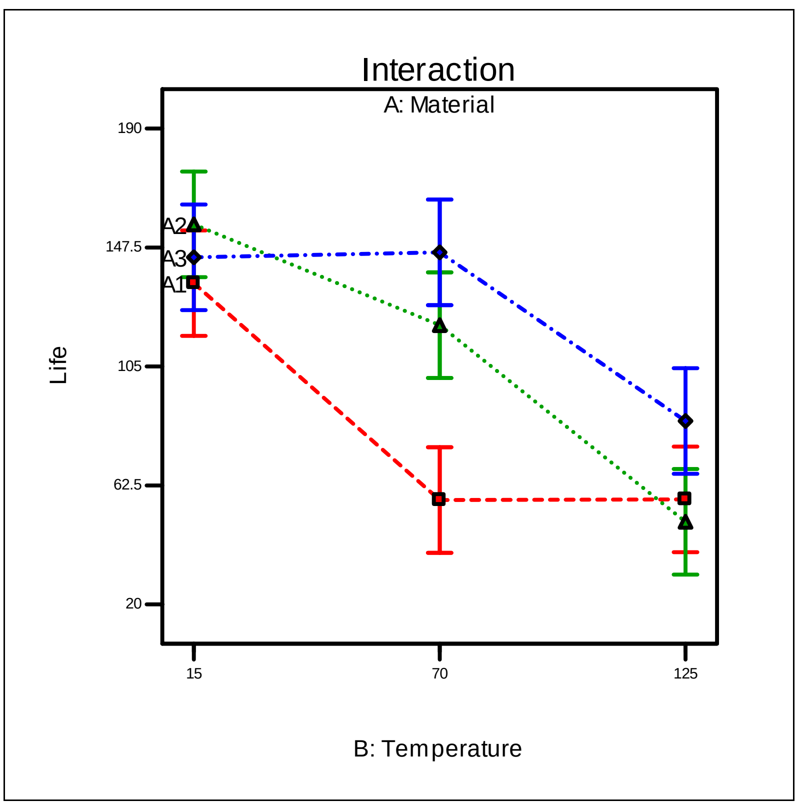
Right-click menu selection for graph preferences

Then click the **Graphs 1** tab and turn off the **Show design points on graph** option.



Turning off design points

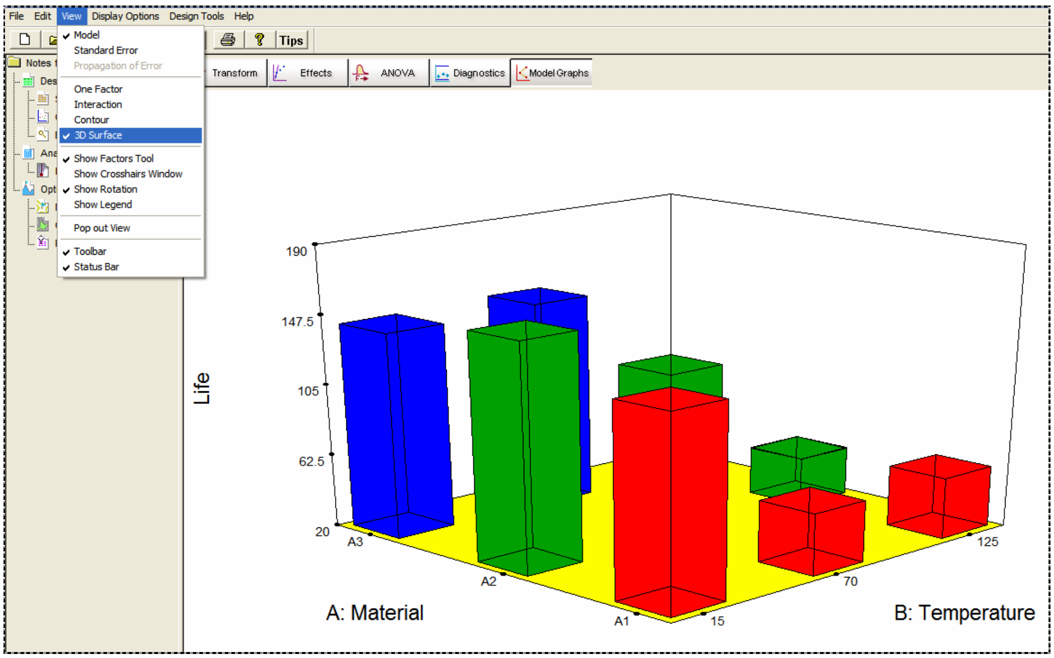
Press **OK**. Your screen should now look like that shown below produced via Edit, Copy from Design-Ease and Edit, Paste in Microsoft Word. (We chose the "Thick" lines for this figure so it would print better in color.)



Clean-looking interaction graph

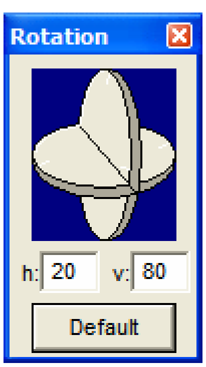
From this graph you can see that all three materials work very well at the low temperature (15 degrees). Based on the overlapping LSD bars, it would be fair to say that no material stands out at this end of the scale. However, the A1 material clearly falls off at the 70 degree temperature, which would be encountered most often, so it must be rejected. None of the materials do very well at the highest temperature (125 degrees), but the upper end of the LSD bar for A2 barely overlaps the bottom end of the LSD bar for A3. Therefore, in respect to temperature sensitivity, material A3 may be the most robust material for making batteries.

Finally, if you do have an opportunity to present graphics in color, here's a dazzling new way to display general factorial effects with Design-Ease: **View**, **3D Surface**.



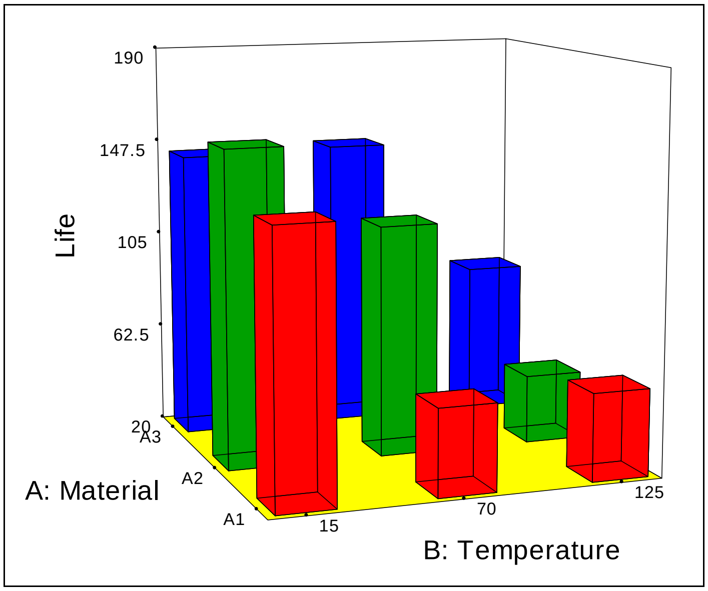
3D surface plot

Place your mouse cursor on the **Rotation** wheel and spin the graph so the temperature axis is at the bottom, or enter coordinates of h (horizontal) 20 and v (vertical) 80.



Rotation tool

Your graph should now look like the one copied from Design-Ease and pasted below.



3D surface graph

The 3D view presents a different perspective of the general factorial effects – more on a macro level of the overall experimental landscaped. Now the inferiority of material A1 (red bars) becomes obvious: The other two materials tower over it at the mid-temperature of 70 degrees F. Clearly the next step is to eliminate material A1 from contention and perhaps do some further investigation on A2 and A3.