Section 9 – Statistical Details: Analysis

This section of the manual provides specific details on selected aspects of the statistical analyses you see in Design-Ease® software outputs. The primary source of information remains the on-line Help system. Always check Help first before the manual. In this section and throughout the manual, we've provided references for follow-up by those who really want to know where the numbers come from. See the Appendix for a comprehensive list, including the references noted below.

If you need more background on statistical modeling, we recommend:

- *Applied Regression Analysis* by Draper and Smith
- *Applied Linear Regression* by Weisberg.

For details on DOE, we suggest these textbooks:

- *Design and Analysis of Experiments* by Montgomery
- *Response Surface Methodology* by Myers and Montgomery

All the books cited above can be obtained from the publisher - John Wiley and Sons, Inc., New York. The latter two references can also be obtained via Stat-Ease.

We also recommend two paperbacks that you can purchase through Stat-Ease:

- *The Experimenter's Handbook* by Kraber, et al (free to registered users)
- *DOE Simplified* by Anderson and Whitcomb (Productivity Press).

These references are not as detailed, but they cover the basics very well.

If you need a thorough DOE education, consider attending one or more of Stat-Ease's computer-intensive workshops. Call us to get information on course content and schedule. We provide complete details on all outputs in these workshops. Also, don't be shy about calling us for statistical help. In many cases this will be provided at no charge, but if this can't be done, you will be advised on our consulting rates. You will find contact information at the end of the Introduction.

Sequence of Analysis

With Design-Ease, you analyze one response at a time by following these steps:

1. If it's a two-level design, choose the model via the full or half-normal plot of effects. For general factorials with pure error estimates, try the Select by

Probability option under the View menu. Otherwise, you must manually designate highest order interactions as error and proceed to the next step.

2. Do analysis of variance (ANOVA) of the overall model and individual coefficients. If any coefficients have p-values above your critical threshold (we suggest 0.1), then go back to step 1, change them to error, and re-do the ANOVA.

3. Inspect various diagnostic plots to statistically validate the model and check for outliers.

4. If the model looks all right, generate model graphs for interpretation: main effect and interaction graphs and the cube plot.

Transformations

Design-Ease provides the user with a broad range of possible response transformations. The appropriate choice depends on subject matter and/or statistical considerations. The software provides extensive diagnostic capabilities to validate statistical assumptions. For further information on interpreting diagnostic data and choosing transformations, refer to Montgomery's *Design and Analysis of Experiments*. The available transformations are:

- Square root
- Log - base e or 10
- Reciprocal square root
- Inverse
- Power of your choice
- Logit (see below)
- Arcsine square root (see below).

The power transformation allows transformation to any power in the range -3 to +3, provided the data are positive. You may add a constant to the data to avoid powers of negative numbers. If the standard deviation associated with an observation is proportional to the mean raised to the alpha power, then transforming the observation by the one minus alpha power gives a scale satisfying the equal variance requirement of the statistical model. As a feature under Diagnostics, Design-Ease offers a helpful plot, called the Box-Cox, which recommends the appropriate power transformation (including the no transformation option). See the section on Two-Level Factorial Tutorials for a case study that demonstrates use of Box-Cox and provides a few details. For more information on this plot, refer to program Help and/or Montgomery's text.

Use the logit transformation when your response varies within a finite range, such as 0 to 100 percent yield. Logit spreads out the values near the boundaries. The transform is:

$$\text{Logit}(Y) = \log_e[(Y - \text{lower limit})/(\text{upper limit} - Y)]$$

The response must be between the lower and upper limit.

The arcsine square root should be used for binomial data, for example, fraction defective. The transform is:

$$Y' = \sin^{-1}\sqrt{Y}$$

For this transformation to be valid, the response data must be in the form of a proportion between zero and one, from samples of equal size.

Design-Ease offers an option to plot the responses in terms of the original response data. It uses the model to calculate the surface matrix of data points and then applies the inverse transform prior to making the plot.

Model Fitting and Hierarchy

The model-fitting step in Design-Ease uses the QR decomposition algorithm (See Chap. 9, *Linpak User's Guide*, J.J. Dongarra, C.B. Moler, J.R. Bunch, G.W. Stewart, 1979, Siam) on the design matrix (X) to compute model coefficients. All computations are carried out on a standardized version of the design matrix to avoid numerical instabilities as much as possible.

Design-Ease checks the model hierarchy before you can run an ANOVA. For example, if you select an interaction, such as BD, for your model without selecting a main effect "parent" of the interaction, such as B, the resulting model would not be hierarchical. A well-formulated model must include all main effects present in the interactions. If the model fails the hierarchy check, you will be warned and offered a chance to correct this problem. (For details on model hierarchy see "A Property of Well-formulated Polynomial Regression Models", Julio Peixoto, *The American Statistician*, Feb. 1990, Vol. 44, No.1, and John A. Nelder, "The Selection of Terms in Response-Surface Models – How Strong is the Weak-Heredity Principle," *The American Statistician*, Nov. 1998, V52, No. 4.)

Missing Data or Botched Levels

Missing data makes the design unbalanced and non-orthogonal, which introduces undesirable properties to the statistical analysis. However, Design-Ease will still provide a solution to the data that remain but some important effects may be lost to aliasing. The best advice regarding missing data is to run all experiments exactly as planned and get the response data. If you can't do this, be sure to do a design evaluation and pay particular attention to the aliases.

For designs with missing data, or with lost orthogonality due to edited factor levels, Design-Ease uses the method of least squares in a hierarchical fashion to compute the coefficients, building up from a base that contains the intercept plus block effects (if any). Next the main effects are estimated and corrected for the intercept, block effects (if any) and other main effects. Then the coefficients for the two-factor interactions are generated from least squares estimates of the model containing the intercept, block effects (if any), all main effects and all two-factor interactions. If you edit the design to the degree that all the original factorial effects cannot be estimated, the program will estimate as many as possible. When all terms of a given order can not be estimated, a subset is selected using forward stepwise regression. This aliasing process will slow down the computations.

For proper use of the normal probability plot, the effects must have a common error variance. Missing data, or altered independent factors, can cause the variance associated with the estimated effects to differ. Therefore the effects must be adjusted (standardized) to correct for this problem. Design-Ease computes standardized effects by multiplying each coefficient by two, and by the ratio of the standard error of the first coefficient computed (usually A) to that of the standard error of the current coefficient (i):

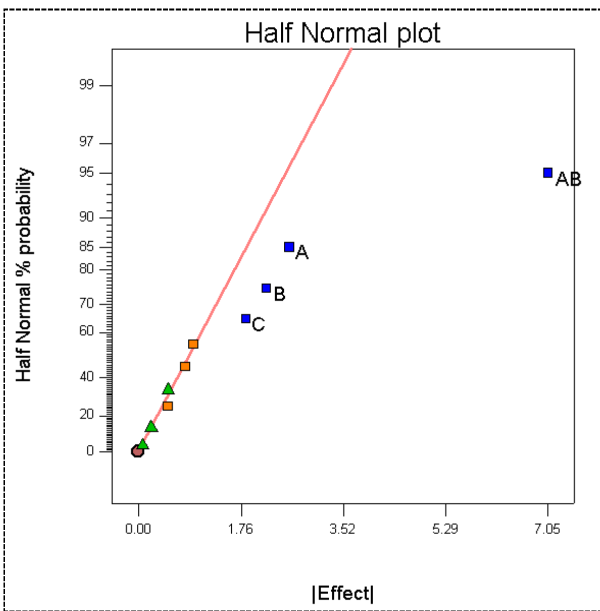
$$\text{Standardized Effect}_i = (\text{Coefficient}_i)(2)(\text{Std. Error}_A/\text{Std. Error}_i)$$

The standard error for effect A is a constant in this equation.

Plotting Pure Error on Effect Plots

Since pure and residual error should each estimate experimental variability, the developers of Design-Ease chose to include the error estimates from replicate points on the probability plots you use to choose model effects. The program adds one pure error effect for each degree of freedom for pure error. These pure error effects are the expected order statistics from a normal sample of size equal to the number of degrees of freedom multiplied by the pure error standard deviation estimate. The pure error effects and factor effects are combined to produce the half normal or full normal probability plot. The pure error points are indicated by "Δ"s.

The normal probability plot for a factorial design with four center points is shown below. Notice the three triangles for the degrees of freedom for pure error.



Pure Error Represented on the Half-Normal Probability Plot

The pure error points should fall in line with the insignificant effects near the zero effect level. For details see "Use of Replication in Almost Unreplicated Factorials," Patrick Whitcomb and Kinley Larntz, Transactions of the 1998 Fall Technical Conference (co-sponsored by the Statistics Division of the American Society of Quality (ASQ) and the American Statistical Association (ASA)). A copy of this presentation can be obtained from Stat-Ease.

Least Significant Difference (LSD) Bars on Model Graphs

The least significant difference method makes use of a rearrangement of Student's t-test for comparing two predicted means. The formula for a balanced two-level factorial is:

$$LSD = t_{\alpha=0.05/2} * \sqrt{2} * \hat{\sigma}\sqrt{x_*^T \left(X^T X\right)^{-1} x_*}$$

where the x_* refer to individual rows in the overall X matrix. The t-value is based on a risk level of 0.05 for a two-tailed test with the degrees of freedom for error associated with the standard deviation (Std. Dev.) on the ANOVA table. The comparison depends on standard error of the mean for a given treatment versus the standard error for all other treatment means. Therefore you will see differences in the sizes of the bars for unbalanced designs. The bars narrow as sample size goes up. Look for overlap between pairs of LSD bars. If there is none, then you can say the associated means differ at the 95 percent confidence levels. See the model graph in the One Factor Tutorial for an example.