

Section 6 – Advanced Design Features

This section discusses advanced design topics. Be prepared to give your software a session of vigorous exercise. You will discover many new program muscles that were not used in the preceding tutorials, which we will assume that you've completed. Don't be afraid to take some side-tracks along the way. In fact, you will be encouraged to explore various features encountered as you work through this section.

You will find more information in the Statistical Details: Design Selection section of this manual. Also, check out the on-line help system in the Design-Ease program. If you still need help, then give Stat-Ease a call. Our telephone number is at the end of the Introduction.

Custom Generators for Fractional Factorials

Design-Ease version 6 automatically generates two-level designs that provide maximum resolution of effects, taking into account the degree of fractionation and/or blocking. The software now allows users to over-ride the design selection criteria by entering custom generators. Before showing how this is done, we will provide a short review of fractional factorial design, introducing a relatively new concept called “abberation.”

A standard two-level fractional factorial can be represented as a 2^{k-p} design, where

- “k” is the number of factors studied,
- 2^{k-p} is the number of experiments run,
- and 2^{-p} is the fraction.

A 2^{k-p} fractional factorial is uniquely defined by the (2^p-1) words in the defining contrast. (See “Design and Analysis of Experiments” by Montgomery for background.)

For example, consider a 2^{7-2} fractional factorial. There are seven factors in a total of thirty-two experiments. It is a 2^{-2} , or a one-quarter fraction of a full 2^7 factorial. The defining contrast contains three (2^2-1) words. Using the default factor generators the defining contrast is:

$$I = DEFG = ABCDF = ABCEG$$

The number of letters in a word is called its “word-length.” The shortest word-length in the defining contrast is called the “resolution” of the design. In our example the shortest word has four letters (DEFG), so the resolution is IV.

Design-Ease created the 2^{7-2} fractional factorial design by taking the standard effect matrix for thirty-two runs (2^5) and generating the extra two factors as follows:

$$\begin{aligned}F &= ABCD \\G &= ABCE\end{aligned}$$

However, other choices of factor generators could create an equally good design in terms of resolution, but not as good for estimating factor interactions. For example, you change the factor generators to:

$$\begin{aligned}F &= ABD \\G &= ACE\end{aligned}$$

Which makes the defining contrast:

$$I = ABDF = ACEG = BCDEFG$$

The design is still resolution IV, i.e., the shortest word in the defining contrast is four letters, but now there are two words of length four, rather than the one word that occurred using the default generators. This causes more two-factor interactions to be aliased with each other. Not good! All else being equal in terms of resolution, the design with the smallest number of shortest word-length words is said to exhibit “minimum aberration”, i.e., the least confusion among the lower order interactions.

Design-Ease version 6 automatically chooses designs of minimum aberration, not only for the fraction itself, but also for the blocking schemes. For details see:

1. D. X. Sun, C. F. J. Wu and Y Chen, “Optimal Blocking Schemes for 2^n and 2^{n-p} Designs,” *Technometrics*, August 1997, Vol. 39, No. 3.
2. R. R. Sitter, J Chen and M. Feder, “Fractional Resolution and Minimum Aberration in Blocked 2^{n-k} Designs,” *Technometrics*, Nov. 1997, Vol. 39, No. 4.

Note that the designs chosen by Design-Ease’s minimum aberration criteria may differ from those in standard textbooks. However, as illustrated by the following mini-tutorial, version 6 of Design-Ease offers the option to define your own generators, thus providing a mechanism to match textbook designs.

In Example 9-6 of his text, Montgomery reports an experiment on a five-axis CNC (computer numeric control) machine used to machine a jet turbine impeller. The engineers varied eight factors in 32 experiments (a 2^{8-3} design) in four blocks (spindles). They desired minimum deviation between the actual versus specified blade profile. To reproduce this DOE, choose **File, New Design**. On the **Factorial** tab, with default design of **2 Level Factorial**, click on the square where 8 factors intersects with 32 experiments ($1/8^{\text{th}}$ fraction). Change the number of **Blocks** from 1 to **4**.

2 Level Factorial Design

Design for 2 to 15 factors where each factor is varied over 2 levels. Useful for estimating main effects and interactions. Fractional factorials can be used for screening many factors to find the significant few. The color coding represents the design resolution: Green = Res V, Yellow = Res IV, and Red = Res III.

		Number of Factors													
		2	3	4	5	6	7	8	9	10	11	12	13	14	15
Experiments	4	Full	1/2 Fract.												
	8		Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.								
	16			Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.	1/256 Fract.	1/512 Fract.	1/1024 Fract.	1/2048 Fract.
	32				Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.	1/256 Fract.	1/512 Fract.	1/1024 Fract.
	64					Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.	1/256 Fract.	1/512 Fract.
	128						Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.	1/256 Fract.
	256							Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.	1/128 Fract.
	512								Full	1/2 Fract.	1/4 Fract.	1/8 Fract.	1/16 Fract.	1/32 Fract.	1/64 Fract.

Replicates: 1 Blocks: 1 Center points per block: 0

Buttons: Cancel Continue >>

Selecting the Fractional Factorial Design for Turbine Case Study

Click on the **Continue** button. Montgomery uses different (but equivalent) factor and block generators. Click on **Make generators editable** to make it possible to change the generators to match those used by Montgomery. Enter on your screen what's shown below.

Factor Generators

F =

G =

H =

Blocking Generators

1 =

2 =

Make generators editable

The default generators give a minimum aberration (the highest possible resolution) design. Editing the generators will change the aliasing patterns and may degrade the design's resolution -- edit with care!

Entering Custom Generators

Click on the **Continue** button. Examine the aliases.

8 Factors: A, B, C, D, E, F, G, H

Design Matrix Evaluation for Factorial Reduced 3FI Model

Factorial Effects Aliases

Est. Terms] Aliased Terms

[Intercept] = Intercept

[Block 1] = Block 1 - EH - ABE + ABH - ACG - ADF - BCD - BFG - CEF + CFH
- DEG + DGH

[Block 2] = Block 2 - EH + ABE - ABH - ACG - ADF - BCD - BFG + CEF - CFH
+ DEG - DGH

[Block 3] = Block 3 + EH - ABE - ABH + ACG + ADF + BCD + BFG - CEF - CFH
- DEG - DGH

[Block 4] = Block 4 + EH + ABE + ABH + ACG + ADF + BCD + BFG + CEF + CFH
+ DEG + DGH

[A] = A + BCF + BDG

[B] = B + ACF + ADG

[C] = C + ABF + DFG

[D] = D + ABG + CFG

[E] = E

[F] = F + ABC + CDG

[G] = G + ABD + CDF

[H] = H

Alias Structure for Blocks and Main Effects

Notice that the EH interaction is aliased with blocks. The experimenters decided to assign the factors least likely to interact to this combination. This completes the tutorial portion of this case study, which demonstrates how to create your own generators. Press Cancel several times if you'd like to quit now. However, if you'd like to complete the study, click **Continue** and enter the eight factors as shown below.

	Name	Units	Type	Low	High
A:	x-Axis shift	0.001 in	Numeric	0	15
B:	y-Axis shift	0.001 in	Numeric	0	15
C:	z-Axis shift	0.001 in	Numeric	0	15
D:	Tool vendor		Categorical	vendor 1	vendor 2
E:	a-Axis	0.001 in	Numeric	0	30
F:	Spindle speed	%	Numeric	90	110
G:	Fixture height	0.001 in	Numeric	0	15
H:	Feed rate	%	Numeric	90	110

Factor Entry

Click **Continue** to enter the response - the standard deviation (**Std Dev**) of the difference in **mils** between the actual versus specified blade profile.

	Name	Units
	Std Dev	mils

Response Entry

Click **Continue** to bring up the design layout. To double-check the alias structure, go to the **Evaluation** node and click the **Results** button. Simulate the response by right clicking on the response column heading, choosing **Simulate response** and selecting the **Turbine.sim** file. Click **OK** to generate the data.

Analyze the data if you like. Here's a few hints:

- Since the response is standard deviation, try a log transformation. On the half-normal plot, don't overlook smaller, but significant effects of G, D and BE.
- There is an outlier, which should be made obvious by the outlier-t diagnostic plot provided by Design-Ease. Find the offending run in the design layout and Toggle Ignore Status. Then re-analyze the data without the outlier.
- Choose the factor levels to minimize standard deviation. Do the end results depend on the transformation and/or whether you ignore the outlier?

To learn more about the final outcome of the turbine case, see the Montgomery text.

Adding Categorical Factors

Version 6 of Design-Ease software offers lots of flexibility for including categorical factors. For example, in the standard two-level designs, or irregular fractions, you can change any factor from numerical to categorical as shown below.

Factors					
	Name	Units	Type	Low	High
A:	A		Numeric	-1	1
B:	B		Numeric Categorical	-1	1

Changing Factor from Numeric to Categorical

This allows you to enter alpha-numeric lows and highs such as "Cheap" versus "Costly." Also, the program now recognizes that the factor cannot be continuously varied, so it would make no sense to allow a centerpoint.

The general factorial, d-optimal (under Factorial tab) and Taguchi designs in Design-Ease default to the categorical type for factors.

Design Evaluation

With Design-Ease's design layout window, you can modify virtually any aspect of your design. Before undertaking your design of experiments, you may change factor levels; add, delete or duplicate experimental runs; or modify block assignments. See the last part of the One Factor Tutorials for a demonstration of design editing.

Whenever you modify a design, we advise that you make use of Design-Ease's unique evaluation capability. Focus on these properties:

- Aliased effects due to lack of unique design points
- Absence of lack of fit test due to absence of replicates
- Absence of curvature test due to lack of center points.
- High leverage points (near one). (The leverage can be reduced by replication of the offending points.)

For a handy checklist with more details on the properties listed above, see the *Handbook for Experimenters* published by Stat-Ease (given free of charge to all registered software users). You can further modify your design if the evaluation looks poor.